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EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN

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THE OCTOBER MEETING IN NEW YORK

The three hundredth meeting of the American Mathematical Society was held at Columbia University, on Saturday, October 29, 1932, extending through the usual morning and afternoon sessions. The attendance included the following ninety-nine members of the Society:

R. L. Anderson, R. G. Archibald, N. H. Ball, J. L. Barnes, A. C. Berry, Archie Blake, H. F. Bohnenblust, A. B. Brown, J. H. Bushey, S. S. Cairns, B. H. Camp, A. D. Campbell, W. R. Church, George Comenetz, H. M. Cox, E. L. Culbreth, Tobias Dantzig, J. L. Doob, Jesse Douglas, Arnold Dresden, Lillian Dundes, H. T. Engstrom, Fay Farnum, T. S. Fiske, W. B. Fite, Edward Fleisher, Tomlinson Fort, R. M. Foster, E. T. Frankel, E. U. Frey, Orrin Frink, C. A. Garabedian, B. P. Gill, O. E. Glenn, Eli Gourin, M. C. Gray, C. C. Grove, Laura Guggenbühl, C. M. Hebbert, Lulu Hofmann, I. O. Horsfall, Harold Hotelling, J. C. Hughes, S. A. Joffe, R. A. Johnson, B. W. Jones, Edward Kasner, L. S. Kennison, J. R. Kline, M. S. Knebelman, B. O. Koopman, A. W. Landers, M. K. Landers, Harry Langman, Solomon Lefschetz, L. A. MacColl, H. F. MacNeish, A. K. Mitchell, Morris Monsky, L. T. Moore, Richard Morris, G. W. Mullins, C. A. Nelson, Oystein Ore, F. W. Owens, H. R. Phalen, E. L. Post, Walter Prenowitz, G. B. Price, H. W. Reddick, R. G. D. Richardson, J. F. Ritt, H. P. Robertson, S. L. Robinson, K. E. Rosinger, H. G. Russell, C. E. Seely, Wladimir Seidel, I. M. Sheffer, Jacob Sherman, L. P. Siceloff, L. G. Simons, James Singer, C. E. Smith, G. Y. Sosnow, M. H. Stone, Paul Szabo, J. M. Thomas, E. W. Titt, C. C. Torrance, J. L. Vanderslice, Oswald Veblen, Leonard Vexler, R. J. Walker, H. S. White, W. A. Wilson, H. P. Wirth, H. M. Wright, Leo Zippin.

There was no meeting of the Council or of the Board of Trustees.

At the request of the Program Committee, Professor Jesse Douglas, of the Massachusetts Institute of Technology, delivered an address entitled *The Problem of Plateau* at the beginning of the afternoon session.

Titles and cross-references to the abstracts of the papers read at the regular sessions follow below. The morning sessions

were held in two sections; the papers numbered 1–11 were read before Section I, Professor Dresden presiding, those numbered 12–39 before Section II, Professor H. S. White presiding, and Professor Douglas's address and the papers numbered 40–43 before a general session in the afternoon, Professor Ritt presiding. Mr. Fialkow was introduced by Professor Kasner, Mr. Perlo by Professor Hotelling, Mr. Leighton by Professor Wall, and Dr. Saks by Professor Tamarkin. The papers whose abstract numbers are followed by the letter *t* were read by title.

1. *Note on cubic surfaces in the Galois fields of order* 2^n, by Professor A. D. Campbell. (Abstract No. 38–11–239.)

2. *Transformations associated with the lines of a cubic, quadratic, or linear complex*, by Dr. I. O. Horsfall. (Abstract No. 38–11–240.)

3. *Translation families of heat curves*, by Mr. George Comenetz. (Abstract No. 38–11–241.)

4. *The geometry of degenerate heat families*, by Mr. Aaron Fialkow. (Abstract No. 38–11–242.)

5. *On teleparallelism in Finsler space* (preliminary report), by Professor A. K. Mitchell. (Abstract No. 38–11–243.)

6. *Three-dimensional manifolds and their Heegaard diagrams*, by Dr. James Singer. (Abstract No. 38–7–188.)

7. *Characterization of the closed 2-cell*, by Dr. Leo Zippin (National Research Fellow). (Abstract No. 38–11–244.)

8. *On unicoherency about a simple closed curve*, by Professor W. A. Wilson. (Abstract No. 38–11–245.)

9. *On the distribution of Student's ratio for samples from non-normal populations*, by Mr. Victor Perlo. (Abstract No. 38–11–246.)

10. *The converse of Spearman's two-factor theorem*, by Professor B. H. Camp. (Abstract No. 38–11–247.)

11. *Properties related to the Borel property and to the closure of derived sets*, by Dr. Selby Robinson (National Research Fellow). (Abstract No. 38–11–256.)

12. *Canonical expressions in Boolean algebra*, by Mr. Archie Blake. (Abstract No. 38–11–248.)

13. *A new theorem in combinatorial analysis with application to a classification of triadic relations*, by Dr. K. E. Rosinger. (Abstract No. 38–11–249.)

14. *On the progressions associated with a ternary quadratic form*, by Mr. E. H. Hadlock. (Abstract No. 38–11–250–*t*.)

15. *The generalized theorem of Stokes*, by Dr. S. S. Cairns. (Abstract No. 38–11–251.)

16. *On the existence and equality of repeated Riemann integrals*, by Mr. J. G. Deutsch. (Abstract No. 38–11–252–*t*.)

17. *Analytic theory of linear q-difference equations*, by Dr. W. J. Trjitzinsky. (Abstract No. 38–11–253–*t*.)

18. *On the convergence and overconvergence of sequences of polynomials of best simultaneous approximation to several functions analytic in distinct regions*, by Dr. Helen G. Russell and Professor J. L. Walsh. (Abstract No. 38–7–173.)

19. *On the distribution of values of bounded analytic functions*, by Dr. Wladimir Seidel. (Abstract No. 38–11–254.)

20. *On the distributions of the zeros of certain analytic functions*, by Mr. L. A. MacColl. (Abstract No. 38–11–255.)

21. *The triangulation of regular topological n-manifolds in* $(n+r)$*-space*, by Dr. S. S. Cairns. (Abstract No. 38–11–257–*t*.)

22. *Solution of the Zarankiewicz problem*, by Dr. E. W. Miller. (Abstract No. 38–9–228–*t*.)

23. *Cross sections of curves in 3-space*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–11–258–*t*.)

24. *The structure of the number of representations function in a binary quadratic form*, by Dr. Gordon Pall. (Abstract No. 38–11–259–*t*.)

25. *A characterization of fields in the calculus of variations*, by Dr. S. B. Littauer and Professor Marston Morse. (Abstract No. 38–11–260–*t*.)

26. *A class of representations of manifolds*, by Dr. C. B. Morrey (National Research Fellow). (Abstract No. 38–11–261–*t*.)

27. *Partial derivates*, by Mr. J. G. Deutsch. (Abstract No. 38–11–262–*t*.)

28. *Functions having the property* (N), by Mr. J. G. Deutsch. (Abstract No. 38–11–263–*t*.)

29. *Continuity and summability for double Fourier series*, by Dr. J. J. Gergen and Dr. S. B. Littauer. (Abstract No. 38–11–264–*t*.)

30. *A set of independent postulates for Principia Mathematica*, by Professor E. V. Huntington. (Abstract No. 38–11–265–*t*.)

31. *On the convergence of continued fractions in which the elements are independent variables* (preliminary report), by Mr. Walter Leighton. (Abstract No. 38–9–227–*t*.)

32. *Integrals over surfaces in parametric form*, by Dr. E. J. McShane (National Research Fellow). (Abstract No. 38–11–266–*t*.)

33. *Parametrizations of saddle surfaces, with application to the problem of Plateau*, by Dr. E. J. McShane (National Research Fellow). (Abstract No. 38–11–267–*t*.)

34. *A theorem on Volterra integral equations of the second kind with singular kernels*, by Dr. W. M. Rust. (Abstract No. 38–9–229–*t*.)

35. *A property of indefinitely differentiable classes*, by Dr. W. J. Trijitzinsky. (Abstract No. 38–11–268–*t*.)

36. *On the approximation of analytic functions by rational functions of best approximation*, by Professor J. L. Walsh. (Abstract No. 38–11–269–*t*.)

37. *The total variation of* $g(x+h)-g(x)$, by Professor Norbert Wiener and Dr. Rosalind C. Young. (Abstract No. 38–9–230–*t*.)

38. *On topological characterizations of the two-dimensional simplex*, by Professor W. A. Wilson. (Abstract No. 38–11–270–*t*.)

39. *On some functionals*, by Dr. Stanislas Saks. (Abstract No. 38–11–272–*t*.)

40. *Families of heat surfaces*, by Professor Edward Kasner. (Abstract No. 38–11–271.)

41. *On the derivation of necessary conditions for the problem of Bolza*, by Professor G. A. Bliss and Dr. I. J. Schoenberg. (Abstract No. 38–9–223–*t*.)

42. *Jordan measure and Riemann integration*, by Professor Orrin Frink. (Abstract No. 38–9–226.)

43. *Analysis of a multivariate complex into principal components*, by Professor Harold Hotelling. (Abstract No. 38–11–272.)

TOMLINSON FORT, *Associate Secretary*

THE THANKSGIVING MEETING IN AMES

The three hundred first regular meeting of the Society was held at Iowa State College on Friday and Saturday, November 25–26, 1932. About eighty persons attended the meeting, among whom were the following forty-six members of the Society:

E. S. Allen, J. V. Atanasoff, R. W. Babcock, R. P. Baker, M. A. Basoco, W. C. Brenke, E. W. Chittenden, J. T. Colpitts, N. B. Conkwright, A. T. Craig, M. E. Daniells, H. P. Doole, J. M. Earl, C. W. Emmons, M. G. Gaba, Sister M. de L. Gough, Cornelius Gouwens, F. S. Harper, J. C. Hempstead, G. A. Herr, J. J. L. Hinrichsen, D. L. Holl, Louis Ingold, M. H. Ingraham, Dunham Jackson, H. E. Jordan, D. E. Kearney, J. V. McKelvey, M. M. McKelvey, H. W. March, U. G. Mitchell, I. F. Neff, T. A. Pierce, H. R. Pyle, B. L. Remick, H. L. Rietz, P. G. Robinson, L. L. Runge, M. F. Schmeiser, E. R. Smith, G. W. Smith, H. F. Smith, E. B. Stouffer, L. E. Ward, J. J. Wheeler, Roscoe Woods.

On Friday afternoon, Professor H. W. March gave an address entitled *Applications of the theory of elasticity to wood, a material of anisotropic structure.* On Saturday morning, Professor E. B. Stouffer spoke to the Society on *Some canonical forms and their associated geometries.*

On Friday evening a dinner was held at the Memorial Union. About seventy-five persons attended the dinner. The toastmaster was Professor J. V. McKelvey, who first called upon Professor E. R. Smith to give a welcoming address. He then in turn called on Dean R. W. Babcock and Professors W. C. Brenke, U. G. Mitchell, H. L. Rietz, and Dunham Jackson, each of whom gave short addresses. The concluding address was by Dean Charles Friley of Iowa State College who spoke on the relationship of mathematics to science. Immediately preceding the speakers, a musical program was presented by Professor E. S. Allen, and Messrs. C. C. Hurd and C. P. Wells.

On Saturday morning, upon motion of Dean Babcock, the thanks of those present were given to the local committee for the splendid arrangements made for the meeting, and to the invited speakers for their interesting addresses.

Vice-President E. W. Chittenden presided at the meeting on Friday afternoon, and Professor E. R. Smith during the address of Professor March on Friday afternoon. Professor Louis Ingold presided on Saturday morning.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by *t* were read by title. Mr. Brandner was introduced by Professor H. L. Rietz, Mr. Montgomery by Professor E. W. Chittenden, Mr. Robertson by Professor E. S. Allen, and Mr. Wells by Professor J. V. Atanasoff.

1. *Distributions of functions of middle items*, by Dr. A. T. Craig. (Abstract No. 38–11–274.)

2. *A test of the significance of the difference of the correlation coefficients of normal bivariate samples*, by Mr. F. A. Brandner. (Abstract No. 38–11–275.)

3. *A generalized definition of probability*, by Professor E. S. Allen. (Abstract No. 38–11–276.)

4. *Sections of point sets*, by Mr. Deane Montgomery. (Abstract No. 38–11–277.)

5. *The conditions for conformality in the elliptic and hyperbolic geometries*, by Professor H. R. Pyle. (Abstract No. 38–11–278.)

6. *On the reduction of a matrix to its rational canonical form*, by Professor M. H. Ingraham. (Abstract No. 38–11–279.)

7. *The fractional differentiation operator*, by Mr. Fred Robertson. (Abstract No. 38–11–288.)

8. *A technique for the study of the interaction of economic series*, by Professor H. T. Davis. (Abstract No. 38–11–280–*t*.)

9. *Certain irregular non-homogeneous linear difference equations*, by Professor David Moskovitz. (Abstract No. 38–11–281–*t*.)

10. *Mutual impedance of long grounded wires when the conductivity of the earth varies exponentially with depth*, by Dr. Marion C. Gray. (Abstract No. 38–11–282–*t*.)

11. *The deflection of a rectangular plate, supported at two opposite edges, due to a point load*, by Professor D. L. Holl. (Abstract No. 38–11–283.)

12. *On the summability and generalized sum of the series of Legendre polynomials* $\sum n^p X_n(x)$, by Professor W. C. Brenke. (Abstract No. 38–11–284.)

13. *Problems of approximation with integral auxiliary conditions*, by Professor Dunham Jackson. (Abstract No. 38–11–285.)

14. *Solution of Dirac's equation without specialization of the operators*, by Professor J. V. Atanasoff. (Abstract No. 38–11–286.)

15. *Separation of partial differential equations in two unknowns*, by Mr. C. P. Wells. (Abstract No. 38–11–287.)

M. H. INGRAHAM, *Associate Secretary*

ERNEST JULIUS WILCZYNSKI—IN MEMORIAM

1. *Introduction.* Ernest Julius Wilczynski was born in Hamburg, Germany, on November 13, 1876, the son of Max and Friederike (Hurwitz) Wilczynski. He died in Denver, Colorado, on September 14, 1932, having not quite completed his fifty-sixth year. One hardly knows whether to admire most the brilliance of his scientific career, which was prematurely terminated in 1923 by illness, or the fortitude with which he endured physical infirmity for a decade. In his memory and for our own inspiration we here sketch the salient aspects of his life and character, and recall synoptically his many significant contributions to science.

2. *Life.* Wilczynski's early education partly explains what he eventually became. After he had gone to school two years in Hamburg, his family migrated to Chicago, where his father became a naturalized citizen of the United States. The boy Wilczynski attended the elementary schools and the North Division High School of Chicago, completing his high school education in three years. Then with the assistance of an uncle, Ellis Wilczynski of Hamburg, young Wilczynski returned to Germany for the purpose of entering the University of Berlin. Here he studied mathematics, physics, and astronomy under such men as Fuchs, Hensel, Planck, Pringsheim, Schlesinger, Schwarz, and Bauschinger. He received the degree of Ph.D. from the University of Berlin in 1897, being then in his twenty-first year.

Returning to the United States, and failing to secure a position in a university immediately, Wilczynski became temporarily a computer in the Office of the Nautical Almanac at Washington, D.C. It was through the influence of A. O. Leuschner, who had known him at Berlin, that he became an instructor in mathematics at the University of California in 1898. There he was successively instructor, 1898–1902; assistant professor, 1902–06; and associate professor, 1906–07. However, he was abroad as research assistant and associate of the Carnegie Institution of Washington for two years, 1903–05. He was married to Contessa Inez Macola of Verona, Italy, on August 9, 1906. She and three daughters survive him.

Wilczynski was associate professor of mathematics at the University of Illinois, 1907–10. Then he was associate professor of mathematics at the University of Chicago, 1910–14; professor, 1914–26; and professor emeritus from 1926 until his death. His health failed gradually after 1919 but he resolutely continued at his post until early in the Summer Quarter of 1923, when in the midst of a lecture he finally realized that he could go no further and, with a simple statement to that effect, walked from his class-room never to return, leaving his students amazed by the classic self-restraint with which he accepted his tragic fate. It is characteristic of the man, however, that during the long illness which followed he never lost his interest in geometry and never gave up hope and the belief that he would some day be able to return to his academic duties.

Wilczynski received during his lifetime various scientific honors and recognitions, of which the most significant are perhaps the following. He was lec-

turer at the New Haven Colloquium of the American Mathematical Society in 1906, with E. H. Moore and Max Mason. He was at one time Vice-President of the American Mathematical Society; he was for two years Chairman of the Chicago Section, and served for a period as Associate Editor of the Transactions of this society. He was also at one time a member of the Council of the Mathematical Association of America. He won a prize of the Royal Belgian Academy of Sciences in 1909, and was elected a member of the National Academy of Sciences in 1919.

Wilczynski was a master of the difficult art of lucid mathematical exposition. He possessed an elegant style both in spoken and written English. German was his native language, and he was at home with French and Italian. He was such a clear and polished lecturer that he made difficulties seem easy.

Wilczynski's character was gentle, his manner mild. He was unselfish and was habitually thoughtful of others. He was interested in his students and in their success. His friendly, kindly, informal attitude toward his students did much to win their affection and loyalty. He believed that a student could do his best work when he was thoroughly at ease in the presence of the instructor.

The premature termination of Wilczynski's scientific career was a great loss to mathematics. When upon one occasion he spoke in commemoration of the late G. M. Green he used words which can now appropriately be quoted and employed in commemoration of their author:

"In this brief span of years he has won enduring fame. . . . , we mourn in him not the promise of a genius unfulfilled, but the sad untimely loss of a great leader of proven strength whose power and insight had been fully tested, and whose actual achievements can never perish. . . . In his death we have suffered a heavy loss, but his life and work will continue to be, for many of us, an everlasting source of strength and inspiration."

3. *Scientific Work.* Wilczynski began his scientific career as a mathematical astronomer, and published fifteen papers in this field during his lifetime. His interest next turned to differential equations, a subject upon which he published eight papers. He won pre-eminence, however, as a projective differential geometer. In this field he published one book and thirty-two papers. Toward the end of his career he published four papers on functions of a complex variable. Two college texts and fifteen miscellaneous papers bring the total number of his mathematical publications to seventy-seven. This is exclusive of forty-six abstracts of papers presented to the American Mathematical Society, which were published in this Bulletin and which of themselves form an impressive outline of a large part of his work.

A bibliography of Wilczynski's publications follows, references to which will be made by year and number, as for example (1895, 1). It may be observed that, besides private publications, his work appeared in nineteen periodicals and in four languages. We shall discuss the most outstanding of these publications briefly and formulate an estimate of his original contributions to astronomical and mathematical knowledge.

During his student days at Berlin Wilczynski published five papers (1895, 1, 2), (1896, 1, 2), (1897, 1) before his thesis. The first two of these are connected with Schmidt's theory of the sun and are primarily expository in character. The others belong to a series of ten papers closely connected with his thesis (1897, 2) on hydrodynamical investigations of the solar rotation.

Besides the thesis, the reader can get a good idea of Wilczynski's work on astronomy by consulting a popular account (1898, 2) and a more mathematical account (1897, 3). He showed, among other things, that under certain hypotheses the angular velocity of rotation of particles in the sun is the same for all particles equally distant from the axis of rotation. Thus he gave a fairly satisfactory explanation of the interesting fact that the angular velocity of rotation of the sun decreases from the equator to the poles.

Perhaps the most interesting feature of Wilczynski's work on differential equations is that this subject served as a bridge over which his interest crossed the gap between astronomy and geometry. In fact, two of his papers (1901, 1, 2) on differential equations were later included in his book (1906, 1) on the projective differential geometry of curves and ruled surfaces, which established his reputation as a geometer, the first paper being incorporated in Chapter I and the second in Chapter IV.

It has often been stated that Wilczynski was the founder, or inventor, of projective differential geometry. This is not quite precise, for Halphen in the latter part of the nineteenth century was the first ever consciously to undertake and carry to fruition a systematic projective differential investigation. He was primarily interested in curves in the plane and in ordinary space. But Wilczynski was the first ever to appreciate, demonstrate, and exploit the utility of completely integrable systems of linear homogeneous differential equations for projective differential geometry. Thus he created a new method in geometry and established himself as the leader of a new school of geometers, which has been called *the American school* of projective differential geometers. His influence, moreover, was international and was particularly strong in Italy and Czechoslovakia.

Besides studying curves by his own method Wilczynski went on to develop projective differential theories of ruled surfaces in ordinary space, and of surfaces and congruences also in ordinary space. His method can be quite briefly described, but space permits scarcely more than a suggestion of some of his most outstanding geometrical results.

Wilczynski's method consists in associating with each configuration to be studied a certain completely integrable system of linear homogeneous differential equations. He next determines the most general transformation of dependent and independent variables that does not disturb the configuration. Then he carries out this transformation on the system of equations and calculates complete systems of invariants and covariants. He interprets these geometrically, and studies the configuration by means of them.

In such a brief comment as this on Wilczynski's geometrical results much will have to be left unsaid that might be said in a more elaborate discussion. But one can certainly say here that much of the theory of ruled surfaces as it exists today, particularly, what concerns the *osculating and asymptotic reguli* along a generator, the *flecnode curves, flecnode congruence, and flecnode transformation*, and the so-called *principal ruled surface* associated with a given ruled surface, is the work of Wilczynski.

In his prize memoir (1911, 2) he developed a theory of congruences in ordinary space and studied two special types of congruences in some detail, namely, congruences belonging to linear complexes and having Laplace trans-

forms likewise belonging to linear complexes, and secondly congruences whose focal surfaces are quadratics. He introduced (1915, 1) the *axis congruence* and *ray congruence* associated with a conjugate net on a surface in ordinary space, and proved that a conjugate net has equal Laplace-Darboux invariants if, and only if, the ray curves, that is, the curves which correspond on the surface to the developables of the ray congruence, themselves form a conjugate net.

He established the theory of non-developable curved surfaces in a series of five memoirs (1907, 1), (1908, 1, 2), (1909, 3, 4). The theory of the so-called *directrix congruence* originated with him. By means of it and a certain *canonical cubic* and *canonical quadric* he interpreted geometrically the coordinate system which leads to a certain canonical expansion for one non-homogeneous coordinate of an ordinary point on a surface in terms of the other two coordinates. Subsequently he studied (1913, 2) a special class of surfaces now called *coincidence surfaces.*

At one time (1911, 1) he was very much interested in nets of curves in the plane, and when he later studied surfaces on which the directrix curves are indeterminate (1914, 3), he employed to advantage this work on nets of curves.

The problem of finding a geometrical significance of *isothermal conjugacy* of a net of curves was completely solved by him for the first time (1920, 1), using a new notion, namely, the notion of a *pencil of conjugate nets.* He gave a second and even more elegant characterization later (1922, 1), to the effect that a fundamental conjugate net is isothermally conjugate if, and only if, its so-called *flex-ray curves* themselves form a conjugate net.

In the closing years of his activity Wilczynski was concerned with functions of a complex variable. He studied (1919, 2, 3) certain line-geometric representations of a function of a complex variable, and investigated certain projective differential properties of such a function in the last memoirs that he ever published (1922, 3), (1923, 1).

No account of Wilczynski's scientific work should be concluded without giving some prominence to his miscellaneous publications. These range all the way from an essay (1900, 1) on poetry and mathematics, and a popular and philosophical account (1909, 2) of the fourth dimension, to book reviews, college texts (1914, 1), (1916, 1), and commemorations.

The two commemorations that he wrote are one (1902, 1) of a former teacher, Lazarus Fuchs, and another (1919, 3) of G. M. Green of Harvard University, whose already brilliant career was cut short by death at the untimely age of twenty-seven. This commemoration is a beautifully written appreciation of the life, character, and work of Green, for whom Wilczynski had the highest regard, and with whom he had corresponded, although the two men had never met each other personally.

Some of Wilczynski's miscellaneous publications are the result of his active interest in the affairs of the American Mathematical Society. In particular, one (1902, 4) of these in the Bulletin of this Society is a report of the organization and first program of the San Francisco Section, which now no longer exists as such. Another (1909, 1) is a minority committee report.

One of the most important of Wilczynski's activities consisted in directing thesis work. There seems to be no record of the number of master's theses that he directed, but there were many of them. He directed twenty-five doctoral

dissertations, two at the University of Illinois, and twenty-three at the University of Chicago. All of his doctors, except one, survive him and form an influential group of the American mathematical community.

BIBLIOGRAPHY

The following bibliography includes Wilczynski's publications exclusive of abstracts of papers presented to the American Mathematical Society. The following abbreviations taken from Bulletin 63 of the National Research Council will be used.

1. Am. J.: American Journal of Mathematics.
2. Ann. di Mat.: Annali di Matematica Pura ed Applicata.
3. Giorn. di Mat.: Giornale di Matematiche.
4. Math. Ann.: Mathematische Annalen.
5. Pal. Circ. Mat.: Rendiconti del Circolo Matematico di Palermo.
6. Wash. Nat. Ac. Sc. Proc.: Proceedings of the National Academy of Sciences.

The following abbreviations will also be used:

1. An. J.: Astronomical Journal.
2. Ap. J.: Astrophysical Journal.

WILCZYNSKI'S PUBLICATIONS

1895. 1. *Schmidt's theory of the sun*, Ap. J., vol. 1, pp. 112–26.
2. *Notes on Schmidt's theory of the sun*, Ap. J., vol. 2, pp. 69–73.

1896. 1. *Hydrodynamical investigations of the solar rotation*, Ap. J., vol. 3, pp. 101–5.
2. *Outlines of a theory of spiral and planetary nebulae*, Ap. J., vol. 4, pp. 97–100.

1897. 1. *Ueber die Rotationsbewegung der Sonne*, Astronomische Nachrichten, vol. 142, pp. 133–6.
2. *Hydrodynamische Untersuchungen mit Anwendung auf die Theorie der Sonnenrotation*, Mayer and Müller, Berlin.
3. *On the causes of the sun's equatorial acceleration and the sunspot period*, An. J., vol. 18, pp. 57–9.

1898. 1. *Theory of the motion of the spots on Jupiter*, An. J., vol. 19, pp. 61–3.
2. *On the causes of the sun's equatorial acceleration and the sun-spot period*, Publications of the Astronomical Society of the Pacific, vol. 10, pp. 97–9.
3. *On the causes of the sun-spot period*, Ap. J., vol. 7, pp. 124–30.
4. *On the depth of the reversing layer*, Ap. J., vol. 7, pp. 213–4.
5. Nippoldt, *Neue allgemeine Erscheinungen in der täglichen Variation der erdmagnetischen Elemente*, Terrestrial Magnetism, vol. 3, p. 184.

1899. 1. *On an mn^2 parameter group of linear substitutions in mn variables*, Proceedings of the California Academy of Sciences, vol. 1, pp., 59–62.
2. *On systems of multiform functions belonging to a group of linear substitutions with uniform coefficients*, Am. J., vol. 21, pp. 85–106.
3. *On linearoid differential equations*, Am. J., vol. 21, pp. 354–66.
4. *Dynamics of a nebula*, An. J., vol. 20, pp. 67–9.

5. Liznar, *Ueber die Aenderung der erdmagnetischen Kraft mit der Höhe*, Terrestrial Magnetism, vol. 4, pp. 191–2.
6. *A generalization of Appell's factorial functions*, this Bulletin, vol. 5, pp. 388–94.

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2. *On continuous binary linearoid groups and the corresponding differential equations and Λ functions*, Am. J., vol. 22, pp. 191–225.
3. *An application of group theory to hydrodynamics*, Transactions of this Society, vol. 1, pp. 339–52.

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2. *Invariants of systems of linear differential equations*, Transactions of this Society, vol. 2, pp. 1–24.
3. *Geometry of a simultaneous system of two linear homogeneous differential equations of the second order*, Transactions of this Society, vol. 2, pp. 343–62.

1902. 1. *Lazarus Fuchs*, this Bulletin, vol. 9, pp. 46–9.
2. *Reciprocal systems of linear differential equations*, Transactions of this Society, vol. 3, pp. 60–70.
3. *Covariants of systems of linear differential equations and applications to the theory of ruled surfaces*, Transactions of this Society, vol. 3, pp. 423–50.
4. *The first meeting of the San Francisco Section of the American Mathematical Society*, this Bulletin, vol. 8, pp. 429–37.

1903. 1. *Forsyth's Differential Equations*, this Bulletin, vol. 10, pp. 86–93.
2. *On a certain congruence associated with a given ruled surface*, Transactions of this Society, vol. 4, pp. 185–200.

1904. 1. *A fundamental theorem in the theory of ruled surfaces*, Math. Ann., vol. 58, pp. 249–59.
2. *On self-dual scrolls*, this Bulletin, vol. 11, p. 8.
3. *Studies in the general theory of ruled surfaces*, Transactions of this Society, vol. 5, pp. 226–52.
4. *On ruled surfaces whose flecnode curve intersects every generator in two coincident points*, Transactions of this Society, vol. 5, pp. 438–46.
5. *Invariants of a system of linear partial differential equations, and the theory of congruences of rays*, Am. J., vol. 26, pp. 319–60.

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2. *General projective theory of space curves*, Transactions of this Society, vol. 6, pp. 99–133.
3. *The general projective theory of space curves and ruled surfaces*, Verhandlungen des dritten internationalen Mathematiker-Kongresses, Heidelberg, 1904, pp. 361–5.

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1908. 1. *Projective differential geometry of curved surfaces* (Second memoir), Transactions of this Society, vol. 9, pp. 79–120.
2. *Projective differential geometry of curved surfaces* (Third memoir), Transactions of this Society, vol. 9, pp. 293–315.

1909. 1. *Mathematical appointments in colleges and universities*, this Bulletin, vol. 15, pp. 492–5.
2. *The fourth dimension*, American Mathematical Monthly, vol. 16, pp. 149–53.
3. *Projective differential geometry of curved surfaces* (Fourth memoir), Transactions of this Society, vol. 10, pp. 176–200.
4. *Projective differential geometry of curved surfaces* (Fifth memoir), Transactions of this Society, vol. 10, pp. 279–96.
5. Hesse, *Vorlesungen aus der analytischen Geometrie der geraden Linie, des Punktes und des Kreises in der Ebene*, this Bulletin, vol. 15, pp. 186–8.

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2. Schlesinger, *Vorlesungen über lineare Differentialgleichungen*, this Bulletin, vol. 16, pp. 483–9.

1911. 1. *One-parameter families and nets of plane curves*, Transactions of this Society, vol. 12, pp. 473–510.
2. *Sur la théorie générale des congruences*, Mémoires publiés par la Classe des Sciences de l'Académie Royale de Belgique, (2), vol. 3.

1912. 1. *A forgotten theorem of Newton's on planetary motion and an instrumental solution of Kepler's problem*, An. J., vol. 27, pp. 155–6.

1913. 1. *Ricerche geometriche intorno al problema dei tre corpi*, Ann. di Mat., (3), vol. 21, pp. 1–21.
2. *On a certain class of self-projecive surfaces*, Transactions of this Society, vol. 14, pp. 421–43.
3. *Some general aspects of modern geometry*, this Bulletin, vol. 19, pp. 331–42.

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2. Darboux, *Leçons sur les Systèmes orthogonaux et les Coordonées curvilignes*. Deuxième édition, complétée, this Bulletin, vol. 20, pp. 247–53.
3. *Über Flächen mit unbestimmten Direktrixkurven*, Math. Ann., vol. 76, pp. 129–60.
4. *On a certain completely integrable system of partial differential equations*, Am. J., vol. 36, pp. 231–60.
5. Fischer, *Koordinatensysteme*, this Bulletin, vol. 20, pp. 212–3.

1915. 1. *The general theory of congruences*, Transactions of this Society, vol. 16, pp. 311–27.
2. *Conjugate systems of space curves with equal Laplace-Darboux invariants*, Wash. Nat. Ac. Sc. Proc., vol. 1, pp. 59–61.

1916. 1. *College Algebra with Applications*, Allyn and Bacon.
2. *Some remarks on the historical development and future prospects of the differential geometry of plane curves*, this Bulletin, vol. 22, pp. 317–29.

3. *Interpretation of the simplest integral invariant of projective geometry*, Wash. Nat. Ac. Sc. Proc., vol. 2, pp. 248–52.

1917. 1. *Integral invariants in projective geometry*, Pal. Circ. Mat., vol. 42, pp. 128–37.

1918. 1. *Invariants and canonical forms*, Wash. Nat. Ac. Sc. Proc., vol. 4, pp. 300–5.

1919. 1. *On the form of the power series for an algebraic function*, American Mathematical Monthly, vol. 26, pp. 9–12.

2. *Line-geometric representation of a function of a complex variable*, Transactions of this Society, vol. 20, pp. 271–98.

3. *In memory of Gabriel Marcus Green*, this Bulletin, vol. 26, pp. 1–13.

1920. 1. *Geometrical significance of isothermal conjugacy of a net of curves*, Am. J., vol. 42, pp. 211–21.

2. *One-parameter families and nets of ruled surfaces and a new theory of congruences*, Transactions of this Society, vol. 21, pp. 157–206.

3. *A set of properties characteristic of a class of congruences connected with the theory of functions*, Transactions of this Society, vol. 21, pp. 409–45.

1922. 1. *Charakteristische Eigenschaften der isotherm-konjugierten Kurvennetze*, Math. Ann., vol. 85, pp. 208–12.

2. *Some generalizations of geodesics*, Transactions of this Society, vol. 23, pp. 223–39.

3. *Differential properties of functions of a complex variable which are invariant under linear transformations*, Part I, Journal de Mathématiques, (9), vol. 1, pp. 293–435.

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E. P. LANE

SHORTER NOTICES

Principes Géométriques d'Analyse, Deuxième Partie. By Gaston Julia. Paris, Gauthier-Villars, 1932. vii+121 pp.

This book presents a continuation of the first volume of the author dealing with those aspects of the modern theory of functions of a complex variable which are derivable from simple geometrical principles. As the author himself points out in the preface to the first volume, the most important of these principles is the conformal correspondence between two regions of planar character or two Riemann surfaces realized by an analytic function $Z=f(z)$. The first volume is principally devoted to the study of those conformal correspondences which carry a unit surface into a Riemann surface interior to the unit circle. This subject centers about the Lemma of Schwarz and the interpretation of it in terms of the non-euclidean (Lobachevskian) geometry in a circle, indicated for the first time by G. Pick.

The second volume begins with the maximum principle and consists of generalizations of this principle in various directions associated with the names of Lindelöf, Littlewood, Carleman, and others. By a series of easy extensions from the maximum principle the author arrives toward the end of Chapter 1 at the Phragmén-Lindelöf theorem. Some applications are then made to the theory of integral functions. The chapter ends with an important Lemma of Carleman concerning analytic functions $f(z)$ in regions $AOBC$ where $\overline{OA}$ and $\overline{OB}$ are rectilinear segments and ACB is a Jordan arc. The lemma gives a limitation of $|f(z)|$ on the bisector of the angle AOB when the upper bound of $|f(z)|$ on $\overline{OA}$, $\overline{OB}$, and on the arc ACB, and the magnitude of the angle AOB are known.

Chapter 2 is devoted to some classical facts concerning harmonic functions of two variables and the conformal mapping of simply connected regions on a circle. It is rather unfortunate that in connection with the second topic no reference is made to the important work of W. F. Osgood. The author is further led to consider the problem of Dirichlet and in particular the integral of Poisson. One of the methods described for obtaining the integral is that of reducing it to the mean value theorem for harmonic functions. Here again the reference is not wholly adequate since no mention is made of Bôcher, the first to notice the fact. With the mathematical tools thus obtained, the author proceeds to generalize the lemma of Carleman by giving a limitation of $|f(z)|$ valid in every point of the region $AOBC$.

After a brief introduction to the elliptic modular function in Chapter 3, an exposition of the method of Lindelöf follows in Chapter 4. Here the geometric concept underlying the whole work is a relation between two Riemann surfaces S and S_1 which Julia characterizes by saying that S_1 is carried by S. This means that S_1 is a covering surface of S such that every closed curve on S_1 is projected on a closed curve on S. From Lindelöf's fundamental theorem, which contains the lemma of Schwarz as a particular case, a variety of inequalities is developed for functions analytic in a circle. These inequalities, together with

some properties of the elliptic modular functions, then yield the classical theorems of Picard, Landau, and Schottky.

In Chapter 5 the author introduces the notion of convexity of functions and derives the fundamental theorems of Hardy and Littlewood concerning the means of order p of analytic functions. In connection with Theorem IV of this chapter, a somewhat analogous theorem of F. Hausdorff may be mentioned. The book ends with a short account of F. Riesz's theory of subharmonic functions and its principal applications to the integral means of analytic functions.

Of the misprints in the book there is one of sufficient importance to warrant mention. In the statement of Lindelöf's fundamental theorem the hypothesis $z_0=f(x_0)$ (using the author's notation) is omitted.

The book serves the excellent purpose of unifying by means of geometric concepts various branches of the theory of functions which have hitherto been scattered in the literature. The presentation throughout is lucid, rigorous, and elegant.

W. SEIDEL

Richard Dedekind. Gesammelte mathematische Werke. Herausgegeben von Robert Fricke, Emmy Noether, Öystein Ore. Dritter Band. Braunschweig, Vieweg, 1932. 3+508 pp.

This volume concludes the publication of Dedekind's collected works. The high standard of editorial excellence which marked the previous volumes is maintained. There is but one matter for regret: the promised Life by Fricke is not forthcoming, as it apparently had not been written at the time of Fricke's recent death. Instead, however, we have some extremely interesting letters and notes of Dedekind's own.

This volume contains, among other things, the reprints of three classics: items XLVI–XLIX are the famous Supplement XI to Dirichlet's *Zahlentheorie* in its several editions (the last of which are now becoming scarce); item L is *Stetigkeit und irrationale Zahlen*, and item LI is *Was sind und was sollen die Zahlen?* The careful, complete reprints of these great classics alone make the price (M 43.65, bound) of the volume extremely reasonable.

In addition to these there is much more of great historical interest, and much (especially in the theory of groups) that will repay careful study even today. As an historical document giving an insight into Dedekind's manner of facing and overcoming fundamental difficulties, item XLVIII, *Sur la Théorie des Nombres entiers algébriques*, is of particular interest. The account which Dedekind gives of his various inventions may suggest the degree of difficulty his successors may expect in carrying on his work to the next phase, that of the arithmetic (if there is any) of a non-commutative ring. The problems do not seem even to have been clearly formulated yet. To make the basic advances in algebraic numbers which he did, Dedekind was forced to break essentially new ground. A slavish adherence to his methods, which were created specifically for his problem, rather than an emulation of his pioneering spirit, seems unlikely —from present indications—to lead anywhere in particular.

In conclusion, this splendid edition of Dedekind's works will, let us hope, find its way into every mathematical library.

E. T. BELL

Georg Cantor. Gesammelte Abhandlungen. Herausgegeben von Ernst Zermelo, nebst einem Lebenslauf Cantors von Adolf Fraenkel. Berlin, Springer, 1932. viii+486 pp.

Cantor's *Mengenlehre*, at least in its elements, is so widely known today that the reader of his collected works will perhaps be more interested in following the remarkable evolution of Cantor's distinctive contribution to mathematics than in re-reading the papers on the theory of aggregates—all of which, by the way, are readily available in almost any mathematical library.

It may be said at once, in view of this, that it will scarcely be worth the while of any library which is feeling the effects of the depression to indulge in this volume. The price (approximately $12) seems too high for 494 pages, only a small fraction of which display technical mathematical printing, and none of which exhibit any really complicated typography. Why the price?

Cantor's first work was in the classical theory of numbers; it shows competence but no marked originality. The first paper (No. 6) to hint at a more critical spirit is the two-page Algebraische Notiz, which aims to give a proof of a fundamental lemma in the Galois theory for which, Cantor says, there is no *proof* apparent in the textbooks. This is a proof of the existence of the basic $n!$ valued function. It might be reproduced with advantage in current texts—if it is not indeed open to objections similar to those which cause some doubters today to reject Cantor's definition of Menge. It seems to me that this short note is the first hint that Cantor was about to leave the beaten path.

There follow the papers on analysis, where the critical spirit is plainly evident, particularly in the memoirs on trigonometric series. Somewhere in this period there seems to have been an explosion which shattered the fairly cautious Cantor of the first period, only to reassemble him into a pioneer. The subsequent history of his mind is so well known that I need not dwell on it.

Zermelo's notes refer briefly to the literature which has grown up around Cantor's theories, with occasional indications of difficulties. On the whole the editing appears to be as impartial as is fitting in collected works.

Fraenkel's "Life" is full of interest, as a human document if nothing else. But, it seems to the reviewer, the biographer might have been more explicit just where he takes refuge in sympathetic generalities. In this instance the facts are not irrelevant to the man's work. Extremists, like Poincaré for instance, may have been led to their unfavorable estimates of Mengenlehre as a contribution to rational thought by contemporary knowledge of the facts, or by gossip about them. However that may be, it seems reasonable to say that historians of mathematics in the future will want to know exactly what happened, for this is one instance where the character of the work and the condition of the mind producing it seem to be inseparably connected. "Indessen kommt es zunächst . . . zu einem geistigen Zusammenbruch bei Cantor im Frühjahr 1884, einer psychischen Erkränkung, deren Erscheinungen sich von nun an bis zu seinem Tode zeitweise wiederholten und ihn mehrmals zwangen, eine Nervenklinik aufzusuchen." This carries "de mortuis nihil nisi bonum" to the limit; science is entitled to the facts—if there are any which would cause the critics of Cantorism to revise their estimates. Ambiguity is not a proper monument to a genius like Cantor.

E. T. Bell

NOTES

At the recent meetings at Atlantic City, the following officers were among those elected by the organizations named: The American Association for the Advancement of Science, President, Professor Henry N. Russell of Princeton, Permanent Secretary, Professor Henry B. Ward of Illinois, Chairman of Section A, Professor C. N. Moore, Secretary of Section A, Professor E. R. Hedrick; American Mathematical Society, President, Professor A. B. Coble, Vice-President, Professor J. W. Alexander, Associate Secretary, Professor J. R. Kine (to replace Professor Tomlinson Fort), Members of the Council, Professor G. D. Birkhoff, G. A. Bliss, L. M. Graves, Einar Hille, G. Y. Rainich; Mathematical Association of America, President, Professor Arnold Dresden, Vice-Presidents, Professors A. A. Bennett and E. B. Stouffer, Members of the Board of Trustees, Professors B. F. Finkel, W. L. Hart, E. V. Huntington, and E. J. Moulton.

Among the foreign scientists who have accepted the invitation of the Century of Progress Exposition for the scientific meetings to be held at Chicago during the week beginning June 19 are Professors Lipôt Fejér of the University of Budapest and Tullio Levi-Civita of the University of Rome. Their lectures will be given before a meeting of this Society to be held during that week. Other scientists from abroad who have accepted invitations are listed in the issue of Science for November 25, 1932.

The Optical Society of America has transferred to the American Institute of Physics the publication of the Review of Scientific Instruments. The Institute will expand this journal, adding a section of Physics News and Views, and will coordinate it with the entire list of its publications. Among the other journals now published by the American Institute of Physics is the Reviews of Modern Physics. The Institute offers this journal to members of the American Mathematical Society at the special rate of three dollars a year, instead of the usual four dollars.

The Institute of International Education has just issued the fourth edition of a publication listing fellowships and scholarships that are available to American students for foreign study under various auspices. Copies may be obtained from the Institute, 2 West 45th Street, New York City.

The Warsaw Society of Sciences and Letters (Towarzystwo Naukowe Warszawskie) celebrated, in November, 1932, the twenty-fifth anniversary of its foundation and at the same time the one hundredth anniversary of the suppression of its predecessor, the Philomathic Society (Towarzystwo Przyjaciol Nauk). The latter Society was founded in 1800 and suppressed by the Russian government in 1832.

The following officers and new members of the Council of the London Mathematical Society have been elected for the session 1932–33: President, Professor A. C. Dixon; Vice-Presidents, Professors S. Chapman and H. Levy,

Mr. T. L. Wren; Treasurer, Dr. A. E. Western; Librarian, Professor H. Hilton; Secretaries, Professor G. N. Watson, Mr. F. P. White; new members of the Council, Professor J. C. Burkill, Mr. W. L. Ferrar, Professors E. A. Milne, G. F. J. Temple, and E. C. Titchmarsh.

The Paris Academy of Sciences has awarded its Grand Prize to Professor Louis de Broglie.

The Planck Medal of the German Physical Society has been awarded to Max von Laue, professor of theoretical physics at the University of Berlin.

The Accademia Pontificia dei Nuovi Lincei announces that it will award its next prize, for which competition closes in November, 1933, for a memoir on systems of solutions of differential equations. Competing memoirs may be in English, French, German, Spanish, Italian, or Latin.

The Reale Istituto Lombardo di Scienze e Lettere has established a prize on the Kramer foundation to be awarded every two years for a memoir on an assigned subject in mathematical physics by an Italian engineer. The subject for 1933 is a résumé of the work from the time of Helmholtz to the present on some question in the resistance of fluids that is of interest in aerodynamics and hydraulics, together with new research on the question.

The DeMorgan medal of the London Mathematical Society has been awarded to Bertrand Russell, for his mathematical work.

The Royal Society of London announces the following awards of medals: the Copley medal to Dr. G. E. Hale, for his work on the magnetic field of the sun; the Rumford medal to Professor F. Haber, for his work in the application of thermodynamics to chemical reactions; the Hughes medal to Dr. J. Chadwick, for his researches in radioactivity.

Dr. Irving Langmuir, of the General Electric Company, Schenectady, has been awarded the Nobel prize in chemistry.

Professor I. A. Barnett, of the University of Cincinnati, lectured on *Infinitesimal transformations in certain types of function spaces* before the Mathematical and Physical Society of Budapest on November 24, 1932, at the invitation of that Society. He also delivered a series of lectures before the Mathematical Institute of the University of Cracow in November.

Professor Edward Kasner, of Columbia University, will deliver a series of public lectures in January and February, 1933, at the Peoples Institute, Cooper Union. He will also speak on *Squaring the circle* over the radio on March 3 (Columbia network, WABC).

Mr. N. F. Mott, lecturer in mathematics at the University of Cambridge, has been elected to the Melville Wills chair of theoretical physics at the University of Bristol.

Mr. James Topping has been appointed lecturer in mathematics at the College of Technology, University of Manchester.

The title of emeritus professor of mathematics in the University of London has been conferred on Professor S. A. F. White, on his retirement from the chair of mathematics at King's College, University of London.

Professor O. K. DeFoe, of the College of the Ozarks, has been appointed professor of mathematics and physics at the St. Louis College of Pharmacy.

Dr. Laura Guggenbühl, of Hunter College, has been promoted to an assistant professorship of mathematics.

Professor E. H. McAllister, of the University of Oregon, has been transferred to a professorship of mathematics at the Oregon State College.

Miss Isabel C. McLaughlin and Mr. P. A. Benitz have been appointed to instructorships in mathematics at Hunter College.

At the University of Chicago Professors E. H. Moore and H. E. Slaught have retired from active service in the Department of Mathematics, with the title of Professor Emeritus. Both have been in the University since it opened its doors in the autumn of 1892. Professor Moore was Acting Head of the Department from 1892 to 1896, and Head from 1896 until 1931 when he gave up his administrative duties. He was one of the founders of the Chicago Section of the American Mathematical Society and one of the first editors of its Transactions. He has been president of the Society and of the American Association for the Advancement of Science, and for many years an Associate Editor of the Proceedings of the National Academy of Sciences. These are but a few of his distinguished services to mathematics and to science in this country. Professor Slaught was a Fellow in Mathematics at the University of Chicago from 1892 to 1894, and was one of the first group of graduate students who received the Ph.D. degree in Mathematics from the University. From 1894 on he has been a member of the staff of the Department of Mathematics. He was a central figure in the founding of the Mathematical Association of America, and has been for many years managing editor of its official journal, the American Mathematical Monthly. He has served the American Mathematical Society with distinction as secretary of the Chicago Section and in numerous other capacities, and has been president of the Mathematical Association. The inspiring significance of his influence in college and university mathematical instruction is widely recognized. For an adequate record of the services of such men as these books, not paragraphs, are needed. [Since this paragraph was first written, Professor Moore died on December 30, 1932. A more adequate notice concerning him will appear in an early issue of this Bulletin.]

Dr. J. von Neumann has been appointed professor of mathematical physics at Princeton University.

Dr. Ruth W. Stokes has been appointed associate professor of mathematics at the North Texas State Teachers College, Denton.

Professor Oswald Veblen, who, as previously announced, will join the staff of the Institute of Advanced Study at Princeton, has presented his resignation

to the trustees of Princeton University. Professor Veblen has been connected with Princeton for 27 years.

Professor J. W. Alexander, of Princeton University, has accepted an appointment as professor of mathematics at the Institute of Advanced Study at Princeton.

Dr. William Garnett, formerly principal and professor of mathematics in the Durham College of Science, Newcastle-upon-Tyne, died November 1, 1932, at the age of eighty-one.

Professor Takuro Tamaru, professor of theoretical physics in the College of Science of Tokyo Imperial University, died September 22, 1932, at the age of sixty.

Professor R. L. Green, professor emeritus of mathematics at Stanford University, died November 19, 1932, at the age of seventy-one. He had been a member of the American Mathematical Society since 1891.

Professor H. W. Hayward, professor of engineering at the Massachusetts Institute of Technology, died October 19, 1932.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

1. Dr. D. C. Lewis: *Infinite systems of differential equations with applications to partial differential equations.*

Lichtenstein showed (Journal für Mathematik, 1927) how existence theorems for certain non-linear partial differential equations of hyperbolic type with initial and boundary conditions could be made to depend upon existence theorems for certain infinite systems of ordinary differential equations. In this paper analogous results are obtained for partial differential equations of less restricted type and for more general initial conditions. The gain in generality is principally due to a natural extension of the notion of a solution of a partial differential equation. Such extensions have also been considered for linear equations by N. Wiener, and for Laplace's equation by M. Bôcher. The author proposes the further study of a type of infinite systems, apparently not hitherto considered, of which the following system, arising from the consideration of a stretched elastic string constrained to vibrate transversally, is an example: $d^2x_\alpha/dt^2+\partial V/\partial x_\alpha=0$, $\alpha=1, 2, \cdots$, where $V=(1/2)\sum_{\alpha=1}^{\infty}\alpha^2x_\alpha^2+(\pi/48)\cdot(\sum_{\alpha=1}^{\infty}\alpha^2x_\alpha^2)^2+(\pi/96)\sum_{k=1}^{\infty}(\sum_{\alpha=1}^{k-1}\alpha(k-\alpha)x_\alpha x_{k-\alpha}+2\sum_{\alpha=1}^{\infty}\alpha(k+\alpha)x_\alpha x_{k+\alpha})^2$. (Received November 23, 1932.)

2. Dr. C. E. Rhodes: *Concerning the double Poisson integral and its derivatives.*

The single Poisson integral defines a function, $F(r, x)$, throughout the interior of the unit circle, which, under proper conditions, approaches an arbitrary given function, $f(x)$, along the boundary. Fatou has established (Acta Mathematica, vol. 30, pp. 335–400) certain sufficient conditions that the partial derivatives F_x and F_{xx} should approach respectively $f'(x)$ and $f''(x)$, as the point (r, x) approaches the boundary of the circle. The present paper considers the analogous situation for the function $F(r, \rho, x, y)$ defined by the double Poisson integral. Sufficient conditions that the partial derivatives F_x, F_y, F_{xy}, F_{xx}, F_{yy} shall approach respectively f_x, f_y, f_{xy}, f_{xx}, f_{yy}, as r and ρ approach 1, are found. In order to insure the proper behavior of the double integral over a cross-region about the point under consideration, an additional condition is used which has no analogy in the case of the single Poisson integral. (Received November 28, 1932.)

3. Dr. Mildred M. Sullivan: *Newtonian potentials as functionals of the masses.*

A Newtonian potential and its derivatives may be regarded as functionals of position, the distribution bearing surface or volume, and the density of the distribution. In a paper to appear in the January number of the Transactions of this Society, Dr. Sullivan has discussed the continuity of these functionals with respect to position alone, the masses being fixed, and has employed a simple condition due to Dini which is more general than the usual Hölder condition. The present paper, which was written in collaboration with the late Professor Kellogg, consists in an application of this same condition to the question of continuity of the potentials and their derivatives with respect to variations of the densities and the surfaces or volumes involved. (Received November 28, 1932.)

4. Dr. W. J. Trjitzinsky: *A theory of quasi-analytic classes.*

The author considers indefinitely differentiable classes C suitably defined by means of certain general types of infinite series. Application of the Schmidt theory of systems of linear equations in infinitely many unknowns (Palermo Rendiconti, vol. 25 (1908)) allows complete and explicit treatment of certain fundamental questions concerning quasi-analytic classes. Moreover, extension is made to the hitherto not extensively developed theory of quasi-analytic functions of the complex variable. The author was originally set along the road to the above developments by some results, due to Carleman, concerning quasi-analytic functions defined by cosine series. Use is also made of several of the author's earlier papers on quasi-analytic functions (Annals of Mathematics, (2), vol. 30, pp. 526–546; vol. 32, pp. 623–658, pp. 659–685). (Received November 28, 1932.)

5. Mr. L. B. Robinson: *On equations in mixed differences.* Part III.

In two abstracts in this Bulletin (vol. 32, pp. 316 and 590) the author has discussed the equation (A): $u'(x)=\sum_{j=1}^{n}P_j(x)u(a_jx+\delta_j)+Q(x)$, an equation similar to but more general than the differential q-difference equation. When the above mentioned abstracts were published the convergence of the solutions of (A) in the neighborhood of certain values of the δ had not been established by the author. Now let us write (A) in a new form as follows: (B) $u_x'(\delta, x)=\sum_{j=1}^{n}P_j(x)u(\delta, a_jx+b_j\delta)+Q(x)$. We can show that the solution of (B) depends on one arbitrary parameter. Also if $Q(x)$ and the $P(x)$ converge within the circle C then the solution converges so long as x and $a_jx+b_j\delta$ remain in C. (Received November 21, 1932.)

6. Professor Harry Bateman: *Some applications of Murphy's theorem.*

In 1830 Murphy gave a formula expressing the integral of a function between the roots of two equations $F(x)=0$ and $G(x)=0$ as an integral involving the logarithm of the ratio of $F(x)$ to $G(x)$. The present author shows that when this integral is regarded as a contour integral it is useful for the discovery of

integrals with variable limits such that both the integrand and the integral itself satisfy a given linear partial differential equation. The case in which the integrand is not uniform within the contour is also considered. (Received November 22, 1932.)

7. Professor F. R. Bamforth and Mr. H. H. Alden: *Concerning the uniqueness of solution curves of* $dy/dx=f(x, y)$.

The uniqueness of solution curves of $dy/dx=f(x, y)$ through a given interior point of the region of definition of $f(x, y)$ which is assumed to be a real, single-valued function of its arguments has been discussed by many writers in recent years. So far, with the exception of Kunugi and Yosie, all these writers have studied only sufficient conditions for uniqueness. This note is an outgrowth of a search for necessary conditions which do not assume $f(x, y)$ continuous as do those of the above mentioned authors. An example is given which shows that no one of the sufficient conditions which the authors of this note have found in the literature is necessary, and indicates that necessary conditions of simple form, if such exist for a general function $f(x, y)$, lie much deeper than do the sufficient conditions found until now. Furthermore, it is shown that, for the system $dy/dx=f(x, y, z)$, $dz/dx=g(x, y, z)$, the function $f(x, y, z)$ may be discontinuous at every point of its region of definition and yet there may be a unique solution through every point. (Received November 25, 1932.)

8. Dr. Frances T. Cope: *Formal solutions of irregular linear differential equations.*

The existence of a complete set of formal solutions of the general homogeneous linear differential equation with rational coefficients,

$$(1) \qquad a_0(x)y^{(n)}(x) + a_1(x)y^{(n-1)}(x) + \cdots + a_n(x)y(x) = 0, \ (a_0(x) \not\equiv 0),$$

was first proved by Fabry (Thèse, Paris, 1885). The corresponding existence theorem for an equation of the form (1) in which the coefficients are merely required to be formal series in descending powers of $x^{1/p}$, p being a positive integer, is here proved by a method developed by Birkhoff (Acta Mathematica, vol. 54 (1930), pp. 205–246) for the study of the analogous linear *difference* equation. This method consists of first reducing the original question to that of the formal reducibility of such an equation and then showing that any such equation is reducible if the "basic integer" p is properly chosen. A résumé is also given of the fundamental definitions and theorems of the algebra and differential calculus of the formal solutions of such differential and difference equations. Applications of the existence theorem and a proof of the converse are developed. These applications include general theorems on formal reducibility and formal equivalence at infinity, analogous to those given by Birkhoff for the linear difference equation. (Received November 19, 1932.)

9. Dr. C. H. Dix: *The length of closed level curves of a harmonic function.*

Let the level curve C' correspond to the circle C_r under a conformal transformation of the region between two level curves $u(x, y)=c_1$ and $u(x, y)=c_2$

on an annular ring. Let $l(C^r)$ denote the length of C^r. Then we have the following result: $l(C^r)/l(C_r)$ is a convex function of $\log r$. This follows by an application of a theorem of F. Riesz (Acta Mathematica, vol. 48 (1926), p. 338). (Received November 25, 1932.)

10. Professor L. M. Graves: *On the existence of the absolute minimum in problems of Lagrange.*

This note points out that the proofs given in the author's paper *On the existence of the absolute minimum in space problems of the calculus of variations* (Annals of Mathematics, (2), vol. 28 (1927), pp. 153–170) are applicable under suitably *weakened hypotheses* to problems of Lagrange in which the side conditions consist of equations in finite form and differential equations of the special form $y_\alpha' = h_\alpha(x, y_1, \cdots, y_k)$. Calculus of variations problems in which the integrand involves higher derivatives are reducible to Lagrange problems of this type, as are also isoperimetric problems in which the integrands of the integrals to be kept constant depend only on the coordinates. (Received November 15, 1932.)

11. Professor E. L. Mackie: *On minimizing the quotient of two definite integrals.*

In the Annals of Mathematics, (2), vol. 30 (1926) p. 393, the problem was considered of finding an arc which minimizes the quotient of two definite integrals while keeping a third integral constant. This was carried through as a problem of Mayer. In the present paper, for the same initial problem the standard method of the calculus of variations is employed in finding the first necessary condition for the required arc, and the function to which the Euler equation is applied is shown to be a linear combination of the integrand functions of the three definite integrals, the coefficients of these functions being constants. (Received November 23, 1932.)

12. Professor Florence M. Mears: *Summability for absolutely convergent series.*

This paper deals with methods of linear transformation by square matrices. It determines the set A of matrices such that every absolutely convergent series, or associated sequence, is transformed into a convergent sequence; the subset of A for which the sum of the transformed series is equal to the sum of the original series; the set B of matrices such that every absolutely convergent series, or associated sequence, is transformed into a sequence whose corresponding series is absolutely convergent. (Received November 23, 1932.)

13. Professor I. M. Sheffer: *An extension of Leibnitz's formula for product-differentiation to differentiation and integration for all real orders.*

In the Transactions of this Society, vol. 32 (1930), E. L. Post has given an extended theory of "generalized differentiation," working with a general differential operator $f(D)$. Among other results, he has generalized the Leibnitz formula for a product $f(D)u(x)v(x)$. An interesting subcase is that where

$f(D)=D^a$, a any real number, giving the Leibnitz formula for fractional integration and differentiation. This subcase appears to be of sufficient interest in itself to warrant a simplified treatment, independent of Post's general theory; the present note serves this purpose. At least two writers previous to Post have considered this subject, but formally rather than rigorously. (Received November 21, 1932.)

14. Dr. A. H. Smith: *On the summability of derived series of the Fourier-Lebesgue type.*

Bosanquet and Linfoot (Journal of the London Mathematical Society, vol. 6 (1931), pp. 117–126; Quarterly Journal of Mathematics, Oxford series, vol. 2 (1931), pp. 207–229) introduced a regular method of summation which is weaker than that of the Cesàro means of any order $\alpha>0$, and applied it to the Fourier series and its conjugate series of a function of class L. The present paper extends the former method of summation to the case of the rth derived Fourier series of a function of this type and to the case of the first derived Fourier series and its conjugate series when the function is restricted to be of bounded variation. (Received November 29, 1932.)

15. Dr. W. J. Trjitzinsky: *The general case of integro-q-difference equations.*

The author proves the existence (in the complex plane) of analytic solutions of equations of the form $L(y)=V(y)$. Here L is an operator, with analytic coefficients, of the type occurring in linear homogeneous q-difference equations of order n. On the other hand, V is an integral operator of the Volterra type. While the treatment of this problem (for the complex plane) has been given by C. R. Adams when there is only one characteristic equation, the present author treats the general case. This paper is a sequel to a paper by the author on linear q-difference equations, which will appear in the Acta Mathematica. (Received November 28, 1932.)

16. Miss Mabel Griffin: *Invariants of Pfaffian systems.*

Let Ω be the exterior product of r independent Pfaffians, and let M_i be the set comprising all products of Ω by i of the derived forms. Each set M_i is invariant for the corresponding Pfaffian system of equations S. The vanishing of M_1 is known to be the condition for passivity of S. The present paper interprets the vanishing of M_2 by means of the notion of a primitive system of S, that is, a system whose derived system is S. The sets M_i are shown to give rise to certain arithmetical invariants of S. They are also used to express a necessary and sufficient condition that any S be completely separable, that is, be equivalent to r equations expressible in terms of r non-overlapping sets of independent variables. (Received November 29, 1932.)

17. Mr. Y. Y. Tseng: *Expansions according to a given system of functions.*

In this paper a non-Hilbertian space is proposed for study. It is characterized abstractly by a simple set of properties, but it can have more than a

denumerable infinity of dimensions. We impose no restriction on the given system. (1) The functions need not be mutually independent, and (2) the number of functions may be non-denumerable. In a space not known to be complete the Bessel inequality and formula for a best approximation are obtained. We establish the equivalence of Fourier and Parseval equalities. If the space is complete, then, by means of the Fourier coefficients, every function can be expanded according to the given system, provided the function lies in a certain subspace connected with that system. Two well known theorems still hold: the Parseval equality and the Riesz-Fischer theorem. It may be noted that the abstract space considered here includes Hilbert space, E. H. Moore's modular space, and others, as special cases. We make use of some machinery developed in Moore's second theory of general analysis (not yet published), and the method is entirely direct, involving no orthogonalization process. (Received November 29, 1932.)

18. Mr. Y. Y. Tseng: *The characteristic value problem of Hermitian functional operators in a non-Hilbertian space.*

The problem studied in this paper is the one formulated by von Neumann and solved by von Neumann and by Stone for (separable) Hilbert space. However, we consider here, as in the preceding paper (abstract No. 39–1–17) a quaternionic, non-separable, complete abstract space. Equivalent conditions of completeness and compactness of a space are discussed. We have also the useful theorems on orthogonal decomposition and representation of linear functionals. Explicit formulas for projections relative to sum- and intersection-spaces are found. We show that the problem always admits a unique solution, if and only if the operator is self-adjoint or hypermaximal. We establish a number of properties of the spectra, resolvents, separations, and resolutions of the identity. The discontinuities of E_t can be non-denumerable. Orthogonal decompositions of E_t are made with reference to its continuity character. For every self-adjoint operator there exist three orthonormal characteristic systems (including differentials). In terms of these, the initial operator and the identity operator can be expanded into general series by means of strict functional Hellinger integrals. The results given here contain, indeed as special instances, those for self-adjoint operators in Hilbert space and those for modular Hermitian matrices in general analysis. (Received November 29, 1932.)

19. Dr. S. S. Wilks: *A generalized sampling distribution for the multiple correlation coefficient.*

The sampling distribution of the multiple correlation coefficient has been found by R. A. Fisher (Proceedings of the Royal Society, A, vol. 121 (1928), pp. 654–673) in samples of N individuals from an n-variate normal population for the two extreme cases, (1) that in which all of the variates are subject to sampling variation, and (2) that in which the $n-1$ independent variates have values which are free from sampling variation, while the dependent variate is normally distributed. The distributions for (1) and (2) are denoted by (A) and (C) respectively, and were obtained by entirely different methods. In this paper the more general problem is considered in which each of an arbitrary number t

of the independent variates has a set of N values which are constant for all possible samples while the remaining $n-t-1$ independent variates and the dependent variate are normally distributed. The distribution of the multiple correlation coefficient for this case is found and is shown to reduce to Fisher's (A) and (C) distributions respectively when $t=0$ and $n-1$. (Received November 18, 1932.)

20. Professor F. D. Murnaghan: *On the parametric representation of orthogonal and unitary square matrices.*

The matrix formulas $\boldsymbol{U}=\exp(i\boldsymbol{H})$, $\boldsymbol{O}=\exp(-\boldsymbol{S})$, where $\boldsymbol{U}, \boldsymbol{H}, \boldsymbol{O}, \boldsymbol{S}$ are, respectively, unitary, Hermitian, orthogonal, skew-symmetric matrices, are used to give parametric representations of orthogonal and unitary matrices. The case of four-rowed matrices is treated in some detail. (Received November 25, 1932.)

21. Professor Leonard Carlitz: *On a theorem of higher reciprocity for polynomials in a Galois field.*

This paper contains a new proof of the theorem of reciprocity of index p^n-1 expressed by $(Q/P)=(-1)^{\rho\nu}(P/Q)$, where P and Q are primary irreducible polynomials in $GF(p^n)$, of degree ν and ρ respectively (American Journal of Mathematics, vol. 54 (1932), pp. 39–50). The proof is modeled after Zeller's proof of the ordinary quadratic reciprocity theorem. (Received November 28, 1932.)

22. Mr. J. O. Blumberg: *Properties of a certain type of continued fractions.*

Gauss has shown that the quotient of two "consecutive" hypergeometric functions can be easily represented by a continued fraction of the second class. In the present paper we study at some length the special case of Gauss's continued fraction $J_1(2x)/J_0(2x)$. By means of recursion formulas a triangular numerical table is constructed from which we obtain numerous number theory relations connected with this continued fraction. For some of these properties geometric interpretations are presented. Finally we point out corresponding properties of more general cases of Gauss's continued fraction. (Received November 25, 1932.)

23. Professor J. B. Coleman: *The Jacobian algorithm for periodic continued fractions as defining a cubic irrationality.*

In the Jacobian continued fraction algorithm, when the partial quotients become periodic, certain ratios of the convergents approach limits. These limits have been shown to be rational linear functions of the principal root of a characteristic equation of the form $\rho^3-M\rho^2+N\rho-1=0$. In this paper the characteristic equation is proved to be irreducible by means of two types of recursion formulas, in terms of which M and N are expressed. (Received November 22, 1932.

24. Professor B. W. Jones: *The transformations effecting the reduction of positive quaternary quadratic forms.*

To date there has been found no reduction of positive quaternary quadratic forms which defines a *unique* form by inequalities on the coefficients. L. Charve has given inequalities which make the sum of the literal coefficients a minimum for the class. We call such a form "semi-reduced." There are usually many semi-reduced forms in a class of equivalent forms. This paper shows that all transformations taking a semi-reduced form into a semi-reduced form are of one or more of four or five types multiplied on the left by certain automorphs. This brings the problem of reduction within reach. (Received November 23, 1932.)

25. Professor Leonard Carlitz: *Some formulas in additive arithmetic.*

In this paper asymptotic expressions are found for a class of sums of the form $\sum\alpha(n-x^2)$, the sum extending over all positive x, $x^2<n$. The method used is quite elementary. By properly specializing the arithmetic function, and using certain known results, we determine the leading term in the asymptotic expression for the number of positive integral solutions of $n=x^2+m_1+\cdots+m_r$, where p^{s_i} does not divide m_i, and the s_i are arbitrary integers $\geqq 2$. (Received November 28, 1932.)

26. Dr. R. D. James (National Research Fellow): *On Waring's problem for odd powers.*

Let $G(k)$ be the least value of r such that all sufficiently large numbers are sums of r non-negative kth powers. In this paper a new method is used to obtain an upper bound for $G(k)$ when k is odd and $\geqq 5$. These values are considerably better than those previously known. (Received November 26, 1932.)

27. Miss Clarice Van Hook: *Pairs of generators of groups of degree not greater than eight.*

Aside from the cyclic groups of odd order, there are sixteen groups which can be written transitively on eight or fewer letters that cannot be generated by two operators of which one is of order two. Pairs of such generators are given in those cases where they exist for the groups of degree that cannot be written transitively of lower degree. (Received November 25, 1932.)

28. Professor C. G. Latimer: *On the class numbers of a cyclic field and a sub-field.*

Let F be an algebraic field which is cyclic and of degree e with respect to the rational field. Assume that the discriminant of every sub-field, not rational, of F contains a prime factor not a divisor of e. If F_1 is a sub-field of F and if h, h_1 is the number of classes of narrowly equivalent ideals in F, F_1, respectively, it is shown that h is divisible by h_1. (Received November 28, 1932.)

29. Professors Jesse Douglas and Philip Franklin: *A step-polygon of a denumerable infinity of sides which bounds no finite area.*

In this paper the authors construct an example, simpler than that previously given by them (Transactions of this Society, vol. 33 (1931), p. 321), of a Jordan curve in space such that every surface bounded by this curve has infinite area. The curve is made up of a denumerable infinity of straight line segments, each parallel to one of the coordinate axes. (Received November 26, 1932.)

30. Professor A. J. Kempner: *On the summation of infinite series.*

Given a Laurent series $\sum a_n(z-\alpha)^n = f(z)$, convergent for $r_1 < |z| < r_2$, and a rational function $R(n) = P(n)/Q(n)$ with complex (including real) coefficients, then the sum of the series $\sum R(n)a_n(z-\alpha)^n = u(z)$, convergent in $r_1 < |z| < r_2$, is expressible by means of the operator $\overline{D} = z(d/dz)$, from the equation $\sum R(n)a_n(z-\alpha)^n = R(\overline{D})f(z)$. The function $u(z)$ satisfies the differential equation $(P(\overline{D})/Q(\overline{D}))f(z) = u(z)$, $Q(\overline{D})u(z) = P(\overline{D})f(z) = g(z)$, say, and this differential equation can be solved by any one of several methods; or, $R(\overline{D})f(z)$ may be immediately reduced to differentiations and integrations. Throughout, proper precautions must be taken concerning values of n for which $R(n) = \infty$. Incidentally, it is seen that a differential equation $R(\overline{D})u(z) = f(z)$, provided $f(z)$ has an expansion $f(z) = \sum a_n(z-\alpha)^n$, has a solution $u(z) = \sum [R(n)]^{-1} \cdot a_n(z-\alpha)^n$. Similar results hold for Dirichlet series $f(s) = \sum c_n e^{k_n s}$ and $\sum R(k_n)c_n e^{k_n s}$, with the operator $D^2 = d/ds$, and more special results for trigonometric series. Extension to series in more than one variable is feasible. (Received December 29, 1932.)

31. Professor H. B. Phillips: *The potential transformation.*

The ordinary potential of a space distribution may be regarded as a transformation which changes a bounded and integrable space density into a function with continuous first derivatives. In this paper the author shows that when valid for all space such a transformation leaves many important relations of scalar and vector functions unchanged. By the use of integrals instead of the usual derivatives the inverse transformation is defined in a form free from certain restrictions ordinarily imposed to insure the existence of second derivatives. (Received November 26, 1932.)

32. Professor I. M. Sheffer: *An aspect of the theory of linear differential equations.*

It occurred to the author that a method he was using to examine some functional equations might be applicable to a phase of linear differential equation theory. It is proposed to give a treatment of this application. First an existence theorem (in the complex domain) is obtained, not by a method of approximations or use of power series but rather by a direct search for the inverse of the differential operator. The solution obtained is expressed as an infinite series which is of interest, not only because of the character of the re-

gion of convergence (which is the interior of a "polygon"), but also because it displays explicitly (although not in "closed" form) the dependence of the solution on the coefficients of the equation. It is believed that the results concerning the series which defines the solution are new; from these are obtained many conclusions which will be recognized as classic. (Received November 21, 1932.)

33. Dr. I. J. Schoenberg and Dr. Wladimir Seidel: *On linear operations in linear metric spaces.*

F. Riesz (Mathematische Annalen, vol. 69 (1910); Annales de l'École Normale Supérieure, (3), vol. 28 (1911); *Les Systèmes d'Équations Linéaires à une Infinité d'Inconnues*, Paris, 1913) and E. Helly (Wiener Sitzungsberichte, IIa, vol. 121 (1912); Monatshefte für Mathematik und Physik, vol. 31 (1921)) have developed methods with the aim to determine the most general linear operations in some special metric spaces. In his recent book *Théorie des Opérations Linéaires*, Warsaw, 1932, S. Banach develops a general method based on prior results of his own and H. Hahn's (Journal für Mathematik, vol. 157). In the present paper, a method is developed which is an independent synthesis of the older methods of Riesz and Helly. This method is more special in its general aspects than the methods of Banach, and for precisely this reason seems to be well adapted to the problem of the determination of the most general linear operation in special linear metric spaces. It permits us to re-establish in a simple way all the known results in this direction and to derive a score of new ones. (Received November 28, 1932.)

34. Professor J. A. Shohat: *On the continued fractions associated with, and corresponding to,* $\int_a^b [p(y)/(x-y)]dy$.

The author gives a discussion of the asymptotic behavior of the distant elements in the continued fractions associated with, and corresponding to, $\int_a^b [p(y)/(x-y)]dy$ ((a, b) finite), for the case of $p(x)$ vanishing over a set of points $\subset (a, b)$ of *positive measure*. The discussion is based on the relation of certain extremal polynomials to the Tchebycheff polynomial ($=T$-polynomial) the least deviating from zero on a given point set. The results of Faber concerning T-polynomials and the notion of "transfinite diameter" (Fekete) for point sets play an important role in the present discussion. (Received November 18, 1932.)

35. Dr. Hassler Whitney (National Research Fellow): *Analytic extensions of differentiable functions defined on closed sets.*

Let E be euclidean n-space, and A, a closed subset of E. Let $f=f_{0\cdots 0}$ be defined and m-fold differentiable ($m \geqq 0$) in A in the following sense: There are functions $f_{k_1\cdots k_n}$ ($k_1+\cdots+k_n \leqq m$) defined in A and such that if x and x' are in A, then $f_{k_1\cdots k_n}(x') = \sum f_{k_1+l_1\cdots k_n+l_n}(x)(x_1'-x_1)\cdots(x_n'-x_n)/(l_1!\cdots l_n!) + R_{k_1\cdots k_n}(x'; x)$ (in $\sum$, $l_1+\cdots+l_n \leqq m-k_1-\cdots-k_n$), where for every $\epsilon>0$ there is a $\delta>0$ (independent of the point x) such that $|R_{k_1\cdots k_n}(x'; x)| < r^{m-k_1-\cdots-k_n}\epsilon$ if the distance r from x' to x is $<\delta$. Then there is a function F defined in E and having continuous partial derivatives through the mth order there, analytic in $E-A$, and coinciding with f in A. A similar theorem holds if $m=\infty$. (Received December 15, 1932.)

36. Professor W. S. Kimball: *The ellipsoidal viscosity distribution function.*

This paper continues the work of the author and his students in removing probability as the foundation stone of statistical mechanics and substituting Newtonian mechanics and elasticity theory. The geometrical weight method is used, and the distribution is an exact solution of Boltzmann's equation. It involves a parameter that varies with molecular position along free paths, and includes a distribution immediately *before* impact that is the mechanical image of that just *after* impact. Elasticity theory relative to homogeneous strain, simple shear, and the strain ellipsoid is taken over into velocity space. The distribution functions before and after impact are obtained directly from the Maxwell distribution by subjecting its equal probability spheres to a simple shear that yields equal probability ellipsoids, being also the strain ellipsoids of the distribution. Distortion but no expansion in velocity space is involved, whereby the entropy and strain are the same as for equilibrium conditions at that temperature. The elastic modulus is again the simple pressure formula, the isothermal bulk modulus for perfect gas. (Received November 23, 1932.)

37. Dr. G. B. Price: *On the Strömgren-Wintner natural termination principle.*

Poincaré developed the subject of analytic continuation of periodic orbits. Recently it has been considered in two papers by Wintner (Mathematische Zeitschrift, vol. 34 (1931), pp. 321–402). He has proved what we shall call the Strömgren-Wintner natural termination principle for groups of periodic orbits. The only example of the principle offered so far is the restricted problem of three bodies, a problem so complicated that the groups can be studied only by numerical integration of the equations of motion. The present paper furnishes a simple example which can be treated mathematically. Wintner's remarks in the proof of the natural termination principle give the impression that his results are in disagreement with those of Poincaré. The present example sets forth the results of both investigators, and shows that there is no essential disagreement. The difference is in the point of view. Finally, this paper adds further information about a class of dynamical systems on surfaces of revolution previously studied by the author (American Journal of Mathematics, vol. 54 (1932), pp. 753–768). (Received November 21, 1932.)

38. Professor C. H. Forsyth: *Interpolation with rationally centered frequencies.*

One of the greatest obstacles to satisfactory interpolation with equi-spaced frequencies, or what are interpreted as frequencies, has heretofore been the practical necessity of associating each frequency with the middle of the range of the corresponding class. The present author has followed the suggestion that the interpolation formula chosen be employed not only to interpolate but also for a preliminary rational approximation of the most appropriate association, and found corresponding simple modifications for a few of the most used interpolation formulas, which will accomplish the purpose described above. These simple modifications prove to have other interesting and valuable virtues also. (Received November 28, 1932.)

39. Mr. J. M. Clarkson: *Some involutorial line transformations.*

Consider a cubic regulus and a plane pencil of lines. An arbitrary line meets three generators of the regulus and one line of the pencil. These four lines have in general one other transversal. The transformation among the Plücker coordinates of the two transversals is of order three. The invariant complex and the singular elements of the transformation are discussed, and the results considered as of a point transformation in five-space. The transformation determined by the transversals of four generators of a quartic regulus is found. Another involutorial line transformation is determined by joining an arbitrary point in one plane to an arbitrary point in a second plane, there being a harmonic homology given in each plane, and the image of the given line being the join of the points which are the images by the two homologies of the given points. (Received November 21, 1932.)

40. Professor T. R. Hollcroft: *The general web of surfaces and the involution defined by it.*

The general web of surfaces is a triply infinite linear system of surfaces of order n. These surfaces are in (1,1) correspondence with the planes of space. This correspondence defines a general space involution whose characteristics are determined. The web contains two doubly infinite, four infinite, and six finite systems of surfaces satisfying respectively one, two, and three invariant conditions associated with singularities or contacts of the surfaces. The jacobian is the locus of the conic nodes and contacts of the two doubly infinite systems. On the jacobian are four curves, the loci respectively of two conic nodes, one binode, two contacts, one stationary contact. The jacobian also contains six finite sets of points which are respectively sets of three conic nodes, one conic node and one binode, one binode whose axis has four-point contact, three contacts, two contacts of which one is stationary, one tacnodal contact. The characteristics of each of the three curves and the number of surfaces in each of the six finite systems are obtained. Webs of quadrics have been studied previously. Certain characteristics of quadric webs do not result on substituting 2 for n in the general formulas, since quadric webs contain degenerate surfaces, a situation that does not arise for $n>2$. (Received November 28, 1932.)

41. Mr. J. C. Knipp: *Certain relations between the Frégier curve and the evolute.*

All chords of a conic which intercept a right angle at a fixed point P of a conic pass through a common point Q; the latter point is known as the Frégier point, and its locus as P moves around the conic is the Frégier curve. This paper considers the relation of this curve to the evolute. The two curves are found to be tangent in four points, and these points are shown to be exceptional for the Frégier curve in several ways. Special properties of the normals from these points to the corresponding conic are developed. (Received November 22, 1932.)

42. Professor Evelyn Carroll Rusk: *Cremona involutions defined by a pencil of cubic surfaces.*

In the American Journal of Mathematics, vol. 54, pp. 707–717, there is a discussion of the Cremona involution defined by means of a pencil of algebraic surfaces $|F_n|$ of order n, having a straight line d to multiplicity $n-2$ and a simple, non-composite curve $\gamma_{4(n-1)}$ of order $4(n-1)$ as base curves. The present paper is restricted to cubic surfaces with a base line d and a residual γ_8 made up of all possible components. The interest lies in the distribution of the parasitic lines, the reduction of the order of the transformation, and the contact of the various principal surfaces along the line d. The generalization applied to a pencil of algebraic surfaces $|F_n|$ offers no new difficulties. (Received November 28, 1932.)

43. Dr. A. E. Staniland: *On the Segre curved four-space representation of the plane of two complex variables.*

Conspicuous among real representations of the complex plane is that of Segre, which employs as representatives of the complex points the real points of a curved space of four dimensions and order six, S_4^6, within an S_8. In the present paper, the author determines the analytic rotations of S_8, i.e., those rotations which transform surfaces representing analytic functions into surfaces representing analytic functions. These rotations are obtained in finding the rotations of S_8 which leave S_4^6 invariant as a whole. Surprisingly, the two classes of rotations are identical. The author classifies the rotations in two ways. First, they are subdivided into ∞^2 sets of ∞^2 members each, each of these sets being subdivided into ∞^1 sets of ∞^1 members each, the sets being characterized by definite effects upon quadric surfaces representative of the lines $Y=MX$. Second, the group is factored into products of types $T_1T_2T_4$, and $T_1T_2T_4$, where T_1, T_2, and T_4 are subgroups each of which leaves one or more of the coordinate axes invariant, and $\overline{T_1}=RT_1$, R being a pair of reflections, one in each of a certain pair of coordinate planes. (Received November 25, 1932.)

44. Dr. S. S. Cairns: *Polyhedral approximations to regular topological n-manifolds.*

A θ-set, in the present paper, will mean $(n+1)$ points which determine such an n-simplex that the angle between any bounding $(n-1)$-simplex and an edge incident with the opposite vertex exceeds $\theta>0$. Consider the triangulation (σ) of M_n announced in the abstract *On the triangulation of topological n-manifolds in $(n+r)$-space* (presented October 29, 1932) and the corresponding simplicial polyhedron, P_n (loc. cit.). For some positive values (ρ, θ), we can make (σ) arbitrarily fine and still require (a) that the vertices of each n-cell be a θ-set and (b) that $s_1>\rho s_1'$, for any 1-simplexes (s_1, s_1') of P_n. A homeomorphism is defined between M_n and P_n such that any i-cell σ_i of (σ) corresponds to the i-simplex, s_i, of P_n having the same vertices. Furthermore, given any positive ϵ, if (σ) is made sufficiently fine under conditions (a) and (b), then (1) the direction cosines of any tangent i-plane (*The direction cosines of a p-space in a euclidean n-space*, American Mathematical Monthly, November, 1932) to σ_i

differ by less than ϵ from the corresponding direction cosines of s_i, and (2) $PP' < \epsilon s_1$ where (P, P') are corresponding points and s_1 is the shortest 1-simplex of P_n. The n-dimensional volume of M_n is then the limit of that of P_n as the triangulation becomes increasingly fine. (Received November 28, 1932.)

45. Dr. P. M. Swingle (National Research Fellow): *Special types of higher dimensional closed sets.*

In this paper a closed set whose r-dimensional Betti number (R. L. Wilder, this Bulletin, vol. 38, p. 679) is zero is called an r-continuum, being for $r=0$ the ordinary continuum. Among these r-continua a continuous curve receives a generalization in what is called a $(j;r)$-continuum; a simple continuous arc in what is called a special Γ r-arc, where Γ is a special type C of complete cycle; and a simple closed curve in what is called a special simple closed r-continuum. A number of theorems are proved concerning these point sets which are generalizations of results well known for the case $r=0=j$. For instance, every special simple closed r-continuum is divided by a Γ of type C into two special Γ r-arcs having only the closure of Γ in common. Also the following generalization of a fundamental lemma of Knaster and Kuratowski (Fundamenta Mathematicae, vol. 2, p. 210, Theorem 6) is obtained: Let M and N be r-continua such that M contains N and $M-N=M_1+M_2$ separate. Then M_i+N $(i=1, 2)$ is an r-continuum. (Received November 28, 1932.)

46. Dr. L. M. Blumenthal and Mr. G. A. Garrett: *Determinantal theory of d-cyclic and pseudo d-cyclic sets of points.* Preliminary report.

The problem of the characterization of the euclidean n-dimensional space R_n among general semi-metric spaces has been shown by Menger (American Journal of Mathematics, vol. 53 (1931), pp. 737–745) to be intimately connected with the theory of the determinant denoted symbolically with first row 0, 1, and second row 1, $(p_ip_j)^2$, $(i, j=1, 2, \cdots, n)$. The investigations presented in the present report show that the characterization of the spherical n-dimensional space S_n is closely related to the theory of the determinant $|\cos \alpha_{ij}|$, $(i, j=1, 2, \cdots, n)$. For the case of the circle S_1, the several theorems characterizing d-cyclic and pseudo d-cyclic sets of points contained in two recent papers by L. M. Blumenthal (American Journal of Mathematics, vol. 54 (1932), pp. 387–396; 729–738) are expressed as new theorems concerning determinants of the above form. (Received November 21, 1932.)

47. Professor N. H. McCoy: *On complete independence of certain sets of postulates for fields.*

In this paper the author gives a method of combining simple systems so as to obtain a system with any desired character. The complete independence of certain sets of field postulates may then be established by exhibiting explicitly only a small number of systems. By means of only ten systems, a set of seven postulates for fields, essentially those given by Huntington (Transactions of this Society, vol. 4 (1903), p. 33) is shown to be completely independent. A set of five systems showing the ordinary independence of the five postu-

lates of Hurwitz (Annals of Mathematics, vol. 15 (1913–14), pp. 93–100) is also sufficient to establish the complete independence of these postulates. The complete independence of this latter set had been previously proved by Bernstein (Annals of Mathematics, vol. 23 (1921–22), pp. 313–316). (Received November 28, 1932.)

48. Dr. G. A. Hedlund: *Recurrent geodesics on any closed orientable surface of genus one.*

The existence of non-periodic recurrent geodesics on certain open surfaces of negative curvature has been proved by Morse. Birkhoff has indicated how the methods of Morse may be used to prove the existence of non-periodic recurrent geodesics on closed orientable surfaces of genus greater than one and of negative curvature. The property of negative curvature plays an essential part. In the case of a closed orientable surface of genus one, the curvature cannot be everywhere negative. Nevertheless it is possible in this case to prove the existence of a non-denumerable set of non-periodic recurrent geodesics. This result will appear in the Proceedings of the National Academy of Sciences. (Received November 23, 1932.)

49. Professor R. C. Bullock: *Non-conjugate osculating quadrics of a curve on a surface.*

It is the primary purpose of this paper to make a study of the projective differential geometry of a non-conjugate net of curves on a surface in ordinary projective space. This study is made by means of a pair of osculating quadrics, one determined by the tangents to three consecutive curves of one family of the non-conjugate net at a point of a general curve on the surface, and the other defined similarly for the other family of the net. Complete results as to the nature of the curve of intersection of the two quadrics thus associated with each point of the surface are obtained. A new necessary and sufficient condition that a net of curves on the surface be a conjugate net is found. Finally, a method of making a similar study of a net on a surface in metric space of three dimensions is indicated. (Received November 25, 1932.)

50. Professor Dunham Jackson: *The convergence of some non-linear processes of approximation.*

If $f(x)$ is a continuous function of period 2π which is everywhere positive, and if an approximating function is sought in the form $e^{T_n(x)}$, where $T_n(x)$ is a trigonometric sum of the nth order, with the specification that the integral of the square of the error (or of the mth power of the absolute value of the error) over a period shall be a minimum, the approximating function depends non-linearly on its parameters, and the detailed consideration of the problem involves certain complications not previously encountered in problems of similar character. With appropriate supplementary hypotheses, this paper demonstrates the existence of an approximating function satisfying the conditions laid down, for fixed n, and its convergence toward $f(x)$ as n becomes infinite. There is an analogous treatment of the corresponding problem of polynomial approximation. (Received November 17, 1932.)

51. Professor G. C. Evans: *Note on Poincaré's sweeping out process.*

The author removes some of the restrictions imposed in the treatment by de la Vallée-Poussin (Annales de l'Institut Henri Poincaré, 1931) of Poincaré's "méthode du balayage." In particular, no restriction is imposed on the original distribution of mass in the region, except that its total amount be finite, and less restrictive conditions of regularity are obtained. (Received November 15, 1932.)

52. Professor Marston Morse: *The tensor form of boundary problems in the calculus of variations.*

Weierstrass removed the limitations on the calculus of variations implied by the non-parametric form. The limitations implied by the use of a single coordinate system have likewise been removed by the use of tensor analysis, in so far as the Euler, Weierstrass, and Legendre conditions are concerned. It is not so with the general accessory boundary problem involving a parameter. When the problem is defined by means of overlapping coordinate systems, it is by no means a priori clear that such an accessory problem exists, still less that it has any invariant or geometric meaning. In this paper an accessory problem is set up in tensor form for the general boundary problem. It reduces to the ordinary accessory problem in non-parametric coordinates when the coordinates are suitably specialized. Characteristic roots and their indices appear as invariants, and hence may properly be considered as geometric in nature. Such problems as the Sturm-Liouville problems and their extensions in the quantum theory, in this relativistic setting appear as special problems in geometry. (Received November 21, 1932.)

53. Professor Henry Blumberg: *Exceptional sets.*

Beginning with an elementary theorem in point sets, the author derives, through a series of steps that easily suggest themselves, a comprehensive body of theorems relating to arbitrarily given sets and functions and asserting that certain associated sets are exceptional, for example, in the sense of cardinal number (denumerability), or of density (non-density, exhaustibility), or of measure (measure zero). There is thus effected a unification and an extension, along simple lines heretofore unobserved, of many known theorems, such as, for example, one of G. C. Young on Dini derivatives, or the author's general theorem on exceptional denumerable sets in Fundamenta Mathematicae, vol. 16 (1930), p. 18, or W. H. Young's results on the symmetry of arbitrary functions of one or more variables, or various results of the author on general functions involving exceptional sets that are exhaustible or of measure zero. (Received Novermber 25, 1932.)

54. Professor Tibor Radó: *An alternating process in the problem of Plateau.*

The problem of Plateau calls for a surface S: $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, with the following properties: (1) $x(u, v)$, $y(u, v)$, $z(u, v)$ are harmonic for $u^2+v^2<1$; (2) they satisfy there the relations $E=G$, $F=0$; (3) these

three functions are continuous in $u^2+v^2\leq 1$, and the equations $x=x(u,v)$, $y=y(u, v)$, $z=z(u, v)$ carry $u^2+v^2=1$ in a topological way into a given Jordan curve. The alternating process developed in the present paper consists in the construction of a sequence of surfaces which all satisfy (3), and satisfy (1) and (2) alternately. The sequence contains a subsequence converging uniformly toward a solution of the problem of Plateau. (Received November 29, 1932.)

55. Professor D. V. Widder: *The inversion of the Laplace integral and the related moment problem.*

The inversion of the Laplace integral $f(x)=\int_0^\infty e^{-xt}d\alpha(t)$ is analogous to the determination of the coefficients of a power series. In fact the integral reduces to a power series in e^{-x} if the function $\alpha(t)$ is a suitable step-function. There are two classical methods of determining these coefficients, one by means of a contour integral, the other in terms of the derivatives of the function represented. We thus expect two methods of inversion of the given integral. That involving contour integration was discovered by Riemann. That involving the derivatives of $f(x)$ was found by E. L. Post (Transactions of this Society, 1930) for the special case in which $\alpha(t)$ has a continuous derivative. It is the purpose of the present paper to treat the general case, and determine $\alpha(t)$ in terms of $f(x)$ and its derivatives. The explicit nature of the inversion formula makes it a powerful tool in applications of the Laplace integral. The moment problem $\mu_n=\int_0^1 t^n d\alpha(t)$ is treated in an analogous manner, and an explicit determination of $\alpha(t)$ in terms of the successive differences of the sequence μ_n is obtained. This inversion formula seems to be entirely new. (Received November 23, 1932.)

56. Dr. E. F. Beckenbach (National Research Fellow) and Professor Tibor Radó: *Subharmonic functions and minimal surfaces.*

The theory of analytic functions of a complex variable has been a guiding principle in the discussion of minimal surfaces since the time of Weierstrass; particularly, this statement applies to the recent developments in the problems of Plateau. Those theorems that are based on the fact that the product of a finite number of analytic functions is again an analytic function have heretofore escaped generalization. We offer now as a substitute for this the fact that if the coordinate functions $x(u, v)$, $y(u, v)$, $z(u, v)$ of a minimal surface are given in terms of isothermic parameters, then $\log(x^2+y^2+z^2)$ is a subharmonic function. This permits in particular the application of the so-called rotation method of analytic function theory to the theory of minimal surfaces. The discussion applies equally to the case of minimal surfaces in n-dimensional space. (Received November 30, 1932.)

57. Dr. Selby Robinson (National Research Fellow): *Pseudo-transitivity in finite and infinite groups.* Preliminary report.

A permutation group is k-ply pseudo-transitive if any set of k symbols can be sent to every unordered set of k symbols by permutations of the group (Carver and King, this Bulletin, vol. 26 (1920), pp. 319–322). If for any m, $0<m\leq k$, a group of degree greater than $m+k$ has permutations which send

every unordered set of k symbols to every other which has exactly $k-m$ symbols in common with the first, the group is k-ply pseudo-transitive. If a k-ply pseudo-transitive group is infinite or of degree $k+k!$ at least, it contains a permutation which leaves invariant an arbitrary set of k symbols but moves any other given symbol. In the general case, the author has been unable to determine whether k-ple pseudo-transitivity implies pseudo-transitivity of order $k-1$ and hence of all lower orders. The theorem is true for groups that are not doubly transitive and that are infinite or of degree $k+(k-1)!(k-2)$ at least. It is also true if $k \leqq 5$ and the degree is at least $k+(k-1)!(k-2)$. For infinite groups and those of degree $k+(k-1)!(k-2)$ at least, double pseudo-transitivity is implied by pseudo-transitivity of higher order k. For finite groups, triple pseudo-transitivity implies ordinary double transitivity, 4-ple pseudo-transitivity implies triple transitivity if the degree is at least $4+3! \cdot 2=16$, and k-ple pseudo-transitivity implies double transitivity if the degree is at least $k+(k-1)!(k-2)$. (Received November 30, 1932.)

58. Dr. C. H. Harry: *Concerning spaces without local cut points.*

The main result of the present paper is the following theorem: If M is any complete, separable, metric, connected and locally connected space having no local cut point, then each compact subset K of M lies in a closed, compact, connected and locally connected subset H of M which contains no local cut point. Further, if K is connected and $\epsilon>0$ any number, then it is possible to choose H in such a manner that for each point X of H there is a point Y of K as close to X as ϵ. (Received December 1, 1932.)

59. Dr. W. T. Reid: *Note on a preceding paper.*

In a recent paper entitled *A boundary value problem associated with the calculus of variations* (American Journal of Mathematics, vol. 54 (1932), pp. 769–790) the author has given sufficient conditions for the considered differential system to have infinitely many characteristic numbers. Hypotheses (H5$^+$) and (H5$^-$) of that paper are somewhat related to the assumption of normality on every subinterval of the given interval x_1x_2. It is the purpose of the present note to point out that the existence of an infinity of characteristic numbers is obtained if these hypotheses are replaced by the following hypothesis: (H5) If η is an admissible arc which is not identically zero on x_1x_2, then the quantity $2G[\eta]+\int_{x_1}^{x_2}\eta_i(x)K_{ij}(x)\eta_j(x)dx$ is different from zero. In some respects this hypothesis is simpler than the hypotheses (H5$^+$) and (H5$^-$). It is also a condition that is satisfied by the accessory boundary value problem associated with the problem of Mayer in the calculus of variations without making the assumption of normality on every subinterval of x_1x_2. (Received November 30, 1932.)

60. Dr. W. T. Reid: *Analogues of the Jacobi condition for the problem of Mayer in the calculus of variations.*

For the general problem of Mayer in the calculus of variations, Cope (Dissertation, Chicago, 1927) has given an analogue of the necessary condition of Jacobi in terms of the characteristic numbers of a boundary value problem associated with the second variation. Bliss (Annals of Mathematics, vol. 33 (1932), pp. 261–274) has phrased the analogue of Jacobi's condition for the

problem of Bolza in terms of the characteristic numbers of a quadratic form involving only a finite number of variables, instead of in terms of the characteristic numbers of a boundary value problem of the type given by Cope. The same form of the condition has also been used by Hestenes (for abstract, see this Bulletin, vol. 38 (1932), p. 348) in treating the problem of Mayer, and the condition differs in form, therefore, from that originally given by Cope. The present paper considers the relations between these two different phrasings of the analogue of the Jacobi condition for the problem of Mayer. (Received November 30, 1932.)

61. Professor P. R. Rider: *The skewness and kurtosis of observations.*

In a paper published in 1921, Coolidge derived an expression for the expected value of the variance of n independent quantities. He also developed a general formula for the variance of a set of statistical observations which reduces for special cases to the corresponding formula for so-called Bernoulli, Lexis, and Poisson series respectively. The present paper derives formulas for the expected values of the skewness and kurtosis, and shows how the usual formulas for the third and fourth moments of Bernoulli, Lexis, and Poisson series emerge as special cases. (Received November 30, 1932.)

62. Dr. T. S. Peterson: *Linear integral equations of functions of two variables.*

It is the purpose of this paper to consider conditions for the solution of the following linear integral equation: $\bar{y}(\alpha, \beta) = y(\alpha, \beta) + \lambda \int_a^b K(\alpha, \sigma) y(\sigma, \beta) d\sigma + \mu \int_a^b L(\beta, \tau) y(\alpha, \tau) d\tau + \nu \int_a^b \int_a^b M(\alpha, \beta, \sigma, \tau) y(\sigma, \tau) d\sigma d\tau$, and, particularly, the truncated form with $M(\alpha, \beta, \sigma, \tau) \equiv 0$. For λ and μ non-characteristic values of the kernels $K(\alpha, \sigma)$ and $L(\beta, \tau)$ respectively, a necessary and sufficient condition that the above equation with $\bar{y}(\alpha, \beta) \equiv 0$, $M(\alpha, \beta, \sigma, \tau) \equiv 0$ have a solution is that λ and μ satisfy the relationship $\lambda/\bar{\lambda} + \mu/\bar{\mu} = 1$, where $\bar{\lambda}$ and $\bar{\mu}$ are characteristic values of the kernels $K(\alpha, \sigma)$ and $L(\beta, \tau)$ respectively. The explicit solutions to this homogeneous equation are also determined. Certain conditions for the solution of the non-homogeneous equations of the above type are also given along with their explicit form. (Received December 2, 1932.)

63. Miss Lucile Lawrence: *On the Kasner ratio curves.*

The curves considered are a family with parameter r, $0 < r < 1$, each being the limit of a sequence of polygons. The first polygon is the same for all sequences, and is open, consisting of two contiguous line segments having distinct orthogonal projections on the x axis. For a given r, each polygon in a sequence is derived from the preceding by taking points on a side at one rth of the distance from each vertex to the mid-point of each adjacent side, joining these points successively in pairs, thus cutting off the corners in a definite way. It is known that for $r = 1/2$, the limiting curve is a parabola. The present paper proves the following theorems: (1) Except for $r = 1/2$, the third derivative fails to exist at a dense set of points, and the curve is non-analytic in any interval. (2a) For $0 < r < 1/2$, the second derivative, if it exists, equals zero at a dense set of points. (2b) For $1/2 < r \leqq 2/3$, the second derivative, if it exists, becomes

infinite at a dense set of points. (2c) For $2/3<r<1$, the right and left first derivatives, if they exist, are unequal at each point of a dense set of points. (Received December 14, 1932.)

64. Dr. C. B. Morrey (National Research Fellow): *An analytic criterion that a surface possess finite Lebesgue area.*

A surface is said to be non-degenerate if it possesses a representation $x^i=x^i(u, v)$, $i=1, \cdots, N$, on the closed unit circle, such that the $x^i(u, v)$ are not simultaneously constant on any continuum containing two points. The representation $x^i=x^i(u, v)$ is said to be generalized conformal if (i) $x^i(u, v)$ is absolutely continuous in the sense of Tonelli, $i=1, \cdots, N$, (ii) $(x_u^i)^2$ and $(x_v^i)^2$ are summable, $i=1, \cdots, N$, and (iii) $E=G$, $F=0$ almost everywhere, where $E=\sum_{i=1}^N (x_u^i)^2$, $F=\sum_{i=1}^N x_u^i x_v^i$, $G=\sum_{i=1}^N (x_v^i)^2$. From recent work on the area of surfaces by McShane and the author, it is known that the Lebesgue area of surfaces thus represented is given by the usual integral formula. Thus, if we admit only non-degenerate surfaces, the following result of the present paper gives a characterization of surfaces of finite Lebesgue area: *Any non-degenerate surface, S, of finite Lebesgue area can be represented generalized conformally on the closed unit circle, R, any three logically distinct points of the boundary of S being made to correspond to three distinct points of the circumference of R.* (Received December 19, 1932.)

65. Professor A. A. Albert: *On primary normal division algebras of degree eight.*

A normal division algebra is called primary if it is not expressible as a direct product of two normal division algebras neither of degree unity. It is well known that a necessary condition that a normal division algebra be primary is that its degree be a power p^e of a prime p. Also a sufficient condition that an algebra of degree p^e be primary is that it have exponent p^e. The author has also shown that a normal division algebra of degree four is primary if and only if its exponent is four. But it has not been known until now whether primary algebras of degree greater than exponent exist. In the present paper it is proved that there exist normal division algebras that are of degree eight and exponent four, and that are primary. (Received December 19, 1932.)

66. Professor A. A. Albert: *Cyclic fields of degree eight.*

In the present paper the author constructs all cyclic fields of degree eight over any non-modular field F. The construction uses purely algebraic methods and gives as explicit formulas as are well known for the case of cyclic fields of degree four. (Received December 19, 1932.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, February 25, 1933.

Abstracts must be in the hands of the Secretary of the Society, 501 West 116th St., New York City, not later than February 4, 1933. Abstracts received by January 7 will appear in the January issue of the BULLETIN. By invitation of the program committee, Professor F. R. Sharpe will deliver an address entitled *The algebraic theory of involutorial transformations.*

STANFORD UNIVERSITY, March 18, 1933.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than February 25. Abstracts received by that date will appear in the March issue of the BULLETIN.

NEW YORK CITY, April 14-15, 1933.

Abstracts must be received at the Office of the Society, 501 West 116th St., New York City, not later than March 24. Abstracts received by March 6 will appear in the March issue of the BULLETIN. A symposium on algebra is being arranged. The leading speakers will be Professor Oystein Ore, on *Abstract ideal theory and applications*, and Professor J. V. Uspensky, *On the theory of numbers and integral calculus.*

CHICAGO, April 14-15, 1933.

Abstracts must be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than March 25. Abstracts received by March 6 will appear in the March issue of the BULLETIN. Professor C. C. MacDuffee will deliver a symposium lecture on *Matrices with elements in a principal ideal ring.*

R. G. D. RICHARDSON, *Secretary of the Society.*

THIRTY-NINTH SUMMER MEETING, CHICAGO, June 19-24, 1933.

This meeting will be in conjunction with the meeting of the American Association for the Advancement of Science at the Century of Progress Exposition, but most of the actual sessions will be at the University of Chicago. Addresses will be delivered by Professors Lipót Fejér of the University of Budapest and Tullio Levi-Civita of the University of Rome. Other addresses and symposia in which American and foreign speakers will participate are being planned. The Mathematical Association of America will meet during the same week. Titles and abstracts of papers to be presented at this meeting should be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than May 20. Abstracts received before May 6 will appear in the May issue of this BULLETIN.

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1932, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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